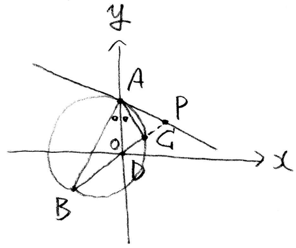
京大理系　1961前期　②



A, D, の座標を $(0,1)$, $(0,0)$ とする

直線 AB, AC, の方程式は $y=\alpha x+1$, $y=-\alpha x+1$ $(\alpha>0)$

直線BCの方程式は $y=\beta x$ $(-\alpha<\beta<\alpha,\ \beta\neq 0)$

$\alpha x+1=\beta x$, $x=\frac{-1}{\alpha-\beta}$, $y=\frac{-\beta}{\alpha-\beta}$, $-\alpha x+1=\beta x$, $x=\frac{1}{\alpha+\beta}$, $y=\frac{\beta}{\alpha+\beta}$ より,

B, C, の座標は $\left(\frac{-1}{\alpha-\beta}, \frac{-\beta}{\alpha-\beta}\right)$, $\left(\frac{1}{\alpha+\beta}, \frac{\beta}{\alpha+\beta}\right)$

外接円の方程式を $(x-X)^2+(y-Y)^2=R^2$ とすると

$X^2+(1-Y)^2=R^2$, $X^2+1-2Y+Y^2=R^2$ —①

$\left(\frac{-1}{\alpha-\beta}-X\right)^2+\left(\frac{-\beta}{\alpha-\beta}-Y\right)^2=R^2$, $\frac{1}{(\alpha-\beta)^2}+\frac{2}{\alpha-\beta}X+X^2+\frac{\beta^2}{(\alpha-\beta)^2}+\frac{2\beta}{\alpha-\beta}Y+Y^2=R^2$ —②

$\left(\frac{1}{\alpha+\beta}-X\right)^2+\left(\frac{\beta}{\alpha+\beta}-Y\right)^2=R^2$, $\frac{1}{(\alpha+\beta)^2}-\frac{2}{\alpha+\beta}X+X^2+\frac{\beta^2}{(\alpha+\beta)^2}-\frac{2\beta}{\alpha+\beta}Y+Y^2=R^2$ —③

②−①より $\frac{1}{(\alpha-\beta)^2}+\frac{2}{\alpha-\beta}X+\frac{\beta^2}{(\alpha-\beta)^2}+\frac{2\beta}{\alpha-\beta}Y-1+2Y=0$, $\frac{1}{\alpha-\beta}+2X+\frac{\beta^2}{\alpha-\beta}+2\beta Y-\alpha+\beta+2(\alpha-\beta)Y=0$

$2X+2\alpha Y+\frac{1+\beta^2+(\alpha-\beta)(-\alpha+\beta)}{\alpha-\beta}=0$, $2X+2\alpha Y+\frac{1+\cancel{\beta^2}-\alpha^2+\alpha\beta+\alpha\beta-\cancel{\beta^2}}{\alpha-\beta}=0$, $2X+2\alpha Y=\frac{\alpha^2-2\alpha\beta-1}{\alpha-\beta}$ —②'

③−①より $\frac{1}{(\alpha+\beta)^2}-\frac{2}{\alpha+\beta}X+\frac{\beta^2}{(\alpha+\beta)^2}-\frac{2\beta}{\alpha+\beta}Y-1+2Y=0$, $\frac{1}{\alpha+\beta}-2X+\frac{\beta^2}{\alpha+\beta}-2\beta Y-\alpha-\beta+2(\alpha+\beta)Y=0$

$-2X+2\alpha Y+\frac{1+\beta^2+(\alpha+\beta)(-\alpha-\beta)}{\alpha+\beta}=0$, $-2X+2\alpha Y+\frac{1+\cancel{\beta^2}-\alpha^2-\alpha\beta-\alpha\beta-\cancel{\beta^2}}{\alpha+\beta}=0$, $-2X+2\alpha Y=\frac{\alpha^2+2\alpha\beta-1}{\alpha+\beta}$ —③'

②'−③'より $4X=\frac{\alpha^2-2\alpha\beta-1}{\alpha-\beta}-\frac{\alpha^2+2\alpha\beta-1}{\alpha+\beta}=\frac{(\alpha^2-2\alpha\beta-1)(\alpha+\beta)-(\alpha^2+2\alpha\beta-1)(\alpha-\beta)}{\alpha^2-\beta^2}=\frac{\cancel{\alpha^3}+\alpha^2\beta-2\alpha^2\beta-\cancel{2\alpha\beta^2}-\cancel{\alpha}-\beta-\cancel{\alpha^3}+\alpha^2\beta-2\alpha^2\beta+\cancel{2\alpha\beta^2}+\cancel{\alpha}-\beta}{\alpha^2-\beta^2}$

$=\frac{-2\alpha^2\beta-2\beta}{\alpha^2-\beta^2}$, $X=\frac{-(\alpha^2+1)\beta}{2(\alpha^2-\beta^2)}$

②'+③'より $4\alpha Y=\frac{\alpha^2-2\alpha\beta-1}{\alpha-\beta}+\frac{\alpha^2+2\alpha\beta-1}{\alpha+\beta}=\frac{(\alpha^2-2\alpha\beta-1)(\alpha+\beta)+(\alpha^2+2\alpha\beta-1)(\alpha-\beta)}{\alpha^2-\beta^2}=\frac{\alpha^3+\cancel{\alpha^2\beta}-\cancel{2\alpha^2\beta}-2\alpha\beta^2-\alpha-\cancel{\beta}+\alpha^3-\cancel{\alpha^2\beta}+\cancel{2\alpha^2\beta}-2\alpha\beta^2-\alpha+\cancel{\beta}}{\alpha^2-\beta^2}$

$=\frac{2\alpha^3-4\alpha\beta^2-2\alpha}{\alpha^2-\beta^2}$　$Y=\frac{\alpha^2-2\beta^2-1}{2(\alpha^2-\beta^2)}$

外接円の中心をZとすると $1-Y=\frac{2\alpha^2-\cancel{2\beta^2}-\alpha^2+\cancel{2\beta^2}+1}{2\alpha^2-2\beta^2}=\frac{\alpha^2+1}{2(\alpha^2-\beta^2)}$ より $\overrightarrow{ZA}=\left(\frac{(\alpha^2+1)\beta}{2(\alpha^2-\beta^2)}, \frac{\alpha^2+1}{2(\alpha^2-\beta^2)}\right)$ より

接線の方程式は $y=-\frac{\frac{(\alpha^2+1)\beta}{2(\alpha^2-\beta^2)}}{\frac{\alpha^2+1}{2(\alpha^2-\beta^2)}}x+1$, $y=-\beta x+1$.

$-\beta x+1=\beta x$, $x=\frac{1}{2\beta}$, $y=\frac{1}{2}$ より Pの座標は $\left(\frac{1}{2\beta}, \frac{1}{2}\right)$

よって、Pは定直線 $y=\frac{1}{2}$ の上にある