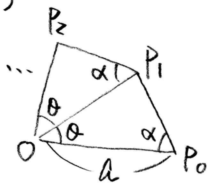
京大理系　1963前期　⑤

(イ)



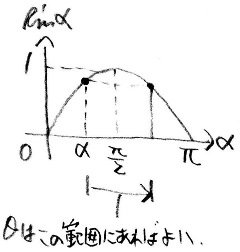
$a = OP_1\cos\theta + P_0P_1\cos\alpha$ —①

$OP_1\sin\theta = P_0P_1\sin\alpha$ —②

①②より. $a = OP_1\cos\theta + OP_1\dfrac{\sin\theta}{\sin\alpha}\cos\alpha$, $a\sin\alpha = OP_1\sin\theta\cos\alpha + OP_1\cos\theta\sin\alpha$.

$a\sin\alpha = OP_1\sin(\theta+\alpha)$. $OP_1 = \dfrac{\sin\alpha}{\sin(\alpha+\theta)}a$ より

$OP_1 = \dfrac{\sin\alpha}{\sin(\alpha+\theta)}a$, $OP_2 = \dfrac{\sin\alpha}{\sin(\alpha+\theta)}OP_1$ … $OP_n = \dfrac{\sin\alpha}{\sin(\alpha+\theta)}OP_{n-1}$ より $OP_n = \left\{\dfrac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^n a$



よって. $n\to\infty$のとき $P_n\to 0$ であるためには. $\sin\alpha < \sin(\alpha+\theta)$ —③ であればよい

左図より. ③が成り立つためには. $0<\alpha<\dfrac{\pi}{2}$ かつ $\theta<\pi-2\alpha$

ところで$\theta>0$であるから. $\theta<\pi-2\alpha$ が成り立つならば $\alpha<\dfrac{\pi}{2}$は常に成り立つ

よって $0<\theta<\pi-2\theta$

(ロ) $OP_0 : OP_1 = 1 : \dfrac{\sin\alpha}{\sin(\alpha+\theta)}$ より $\triangle OP_0P : \triangle OP_1P_2 = 1 : \left\{\dfrac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^2$

$S_n = \triangle OP_0P_1 + \triangle OP_1P_2 + \cdots + \triangle OP_{n-1}P_n$ とすると.

$$S_n = \triangle OP_0P_1\left[1+\left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^2+\cdots+\left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^{2(n-1)}\right] = \frac{1-\left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^{2n}}{1-\left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^2}\triangle OP_0P_1$$

$$S = \lim_{n\to\infty}S_n = \frac{1}{1-\left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^2}\triangle OP_0P_1 = \frac{\sin^2(\alpha+\theta)}{\sin^2(\alpha+\theta)-\sin^2\alpha}\triangle OP_0P_1$$

. よって $\dfrac{\sin^2(\alpha+\theta)}{\sin^2(\alpha+\theta)-\sin^2\alpha}$ 倍

(ハ) $P_1P_2 = \dfrac{\sin\alpha}{\sin(\alpha+\theta)}P_0P_1$, $P_2P_3 = \dfrac{\sin\alpha}{\sin(\alpha+\theta)}P_1P_2$ … $P_nP_{n+1} = \dfrac{\sin\alpha}{\sin(\alpha+\theta)}P_{n-1}P_n$ より. $P_nP_{n+1} = \left\{\dfrac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^n P_0P_1$

①②より $a = P_0P_1\dfrac{\sin\alpha}{\sin\theta}\cos\theta + P_0P_1\cos\alpha$, $a\sin\theta = P_0P_1\sin\alpha\cos\theta + P_0P_1\cos\alpha\sin\theta$, $P_0P_1 = \dfrac{\sin\theta}{\sin(\alpha+\theta)}a$ より

$$P_nP_{n+1} = \left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^n\frac{\sin\theta}{\sin(\alpha+\theta)}a$$

$L_n = P_0P_1 + P_1P_2 + \cdots + P_{n-1}P_n$ とすると.

$$L_n = \frac{\sin\theta}{\sin(\alpha+\theta)}a\left[1+\frac{\sin\alpha}{\sin(\alpha+\theta)}+\cdots+\left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^{n-1}\right] = \frac{1-\left\{\frac{\sin\alpha}{\sin(\alpha+\theta)}\right\}^n}{1-\frac{\sin\alpha}{\sin(\alpha+\theta)}}\frac{\sin\theta}{\sin(\alpha+\theta)}a$$

$$L = \lim_{n\to\infty}L_n = \frac{1}{1-\frac{\sin\alpha}{\sin(\alpha+\theta)}}\frac{\sin\theta}{\sin(\alpha+\theta)}a = \frac{a\sin\theta}{\sin(\alpha+\theta)-\sin\alpha}$$

$$\lim_{\theta\to 0}L = \lim_{\theta\to 0}\frac{a\frac{\sin\theta}{\theta}}{\frac{\sin(\alpha+\theta)-\sin\alpha}{\theta}} = \frac{a}{\cos\alpha}$$