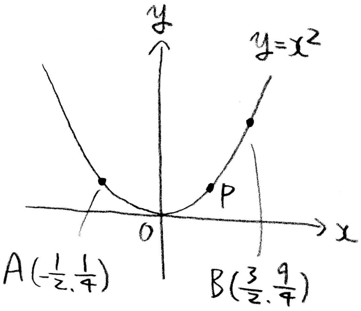
京大理系　1968前期　⑤

$\frac{5}{2}+\frac{41}{8}=\frac{20+41}{8}=\frac{61}{8}$



A, Bの座標は $\left(-\frac{1}{2},\frac{1}{4}\right)$, $\left(\frac{3}{2},\frac{9}{4}\right)$

Pの座標は (x, x^2) とおける.

$$PA^2+PB^2=\left(x+\frac{1}{2}\right)^2+\left(x^2-\frac{1}{4}\right)^2+\left(x-\frac{3}{2}\right)^2+\left(x^2-\frac{9}{4}\right)^2$$

$$=x^2+x+\frac{1}{4}+x^4-\frac{1}{2}x^2+\frac{1}{16}+x^2-3x+\frac{9}{4}+x^4-\frac{9}{2}x^2+\frac{81}{16}$$

$$=2x^4-3x^2-2x+\frac{61}{8}$$

$f(x)=2x^4-3x^2-2x+\frac{61}{8}$ とする.

$4\left(-\frac{1}{8}\right)-3\left(-\frac{1}{2}\right)-1=-\frac{1}{2}+\frac{3}{2}-1=0$

$f'(x)=8x^3-6x-2$.

$f'(x)=0$ のとき. $4x^3-3x-1=0$. $(2x+1)(2x^2-x-1)=0$. $(2x+1)^2(x-1)=0$. $x=-\frac{1}{2}, 1$

$$
\begin{array}{r}
2x^2-x-1 \\
2x+1\,\overline{)\,4x^3\qquad -3x-1} \\
4x^3+2x^2\qquad\quad \\
\hline
-2x^2-3x\quad \\
-2x^2-x\quad \\
\hline
-2x-1 \\
-2x-1 \\
\hline
0
\end{array}
$$

$f''(x)=24x^2-6$

$f''(x)=0$ のとき. $4x^2-1=0$. $x=\pm\frac{1}{2}$

$$
\begin{array}{r}
2x+1 \\
x-1\,\overline{)\,2x^2-x-1} \\
2x^2-2x\quad \\
\hline
x-1 \\
x-1 \\
\hline
0
\end{array}
$$

x	…	$-\frac{1}{2}$	…	$\frac{1}{2}$	…	1	…
$f'(x)$	$-$	0	$-$	$-$	$-$	0	$+$
$f''(x)$	$+$	0	$-$	0	$+$	$+$	$+$
$f(x)$	↘	8	↘	6	↘	$\frac{37}{8}$	↗

$f(x)$の増減表は左表のようになる.

よって, PA^2+PB^2のグラフは右図のようになる.

$f\left(-\frac{1}{2}\right)=\frac{1}{8}-\frac{3}{4}+1+\frac{61}{8}=\frac{31-3+4}{4}=8$

$f\left(\frac{1}{2}\right)=\frac{1}{8}-\frac{3}{4}-1+\frac{61}{8}=\frac{31-3-4}{4}=6$

$f(1)=2-3-2+\frac{61}{8}=\frac{61-24}{8}=\frac{37}{8}$