京大理系 1972前期 ②

$$\frac{1}{t+1}+\frac{1}{t+3}=\frac{t+3+t+1}{(t+1)(t+3)}=2\frac{t}{(t+1)(t+3)}+4\frac{1}{(t+1)(t+3)}$$

$$\frac{1}{t+1}-\frac{1}{t+3}=\frac{\cancel{t}+3-\cancel{t}-1}{(t+1)(t+3)}=2\frac{1}{(t+1)(t+3)}$$

よって $\displaystyle\frac{t}{(t+1)(t+3)}=\frac{1}{2}\frac{1}{t+1}+\frac{1}{2}\frac{1}{t+3}-2\frac{1}{(t+1)(t+3)}=\frac{1}{2}\frac{1}{t+1}+\frac{1}{2}\frac{1}{t+3}-\frac{1}{t+1}+\frac{1}{t+3}=-\frac{1}{2}\frac{1}{t+1}+\frac{3}{2}\frac{1}{t+3}$

$$F(x)=\left[-\frac{1}{2}\log(t+1)+\frac{3}{2}\log(t+3)\right]_0^x=-\frac{1}{2}\log(x+1)+\frac{3}{2}\log(x+3)-\frac{3}{2}\log 3$$

$$F(x)-\log x=-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x+3)^3-\frac{3}{2}\log 3-\frac{1}{2}\log x^2=\frac{1}{2}\log\frac{(x+3)^3}{(x+1)x^2}-\frac{3}{2}\log 3=\frac{1}{2}\log\frac{(1+\frac{3}{x})^3}{1+\frac{1}{x}}-\frac{3}{2}\log 3$$

$$\lim_{x\to+\infty}\left[F(x)-\log x\right]=-\frac{3}{2}\log 3$$