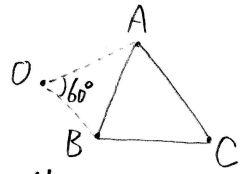
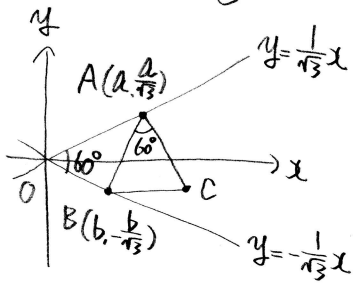
京大理系 1973前期 ③





xy座標で考える.

左図のように O. A. Bをとってよい.

$\overrightarrow{AB}=(b-a,-\frac{b}{\sqrt{3}}-\frac{a}{\sqrt{3}})$ より Cの座標は

$$\begin{pmatrix}a\\ \frac{a}{\sqrt{3}}\end{pmatrix}+\begin{pmatrix}\cos 60^\circ & -\sin 60^\circ\\ \sin 60^\circ & \cos 60^\circ\end{pmatrix}\begin{pmatrix}-a+b\\ -\frac{a}{\sqrt{3}}-\frac{b}{\sqrt{3}}\end{pmatrix}=\begin{pmatrix}a\\ \frac{a}{\sqrt{3}}\end{pmatrix}+\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}-a+b\\ -\frac{a}{\sqrt{3}}-\frac{b}{\sqrt{3}}\end{pmatrix}$$

$$=\begin{pmatrix}a-\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}a+\frac{1}{2}b\\ \frac{\sqrt{3}}{2}a-\frac{\sqrt{3}}{2}a+\frac{\sqrt{3}}{2}b-\frac{\sqrt{3}}{6}a-\frac{\sqrt{3}}{6}b\end{pmatrix}=\begin{pmatrix}a+b\\ \frac{2-3-1}{6}\sqrt{3}a+\frac{3-1}{6}\sqrt{3}b\end{pmatrix}=\begin{pmatrix}a+b\\ -\frac{a}{\sqrt{3}}+\frac{b}{\sqrt{3}}\end{pmatrix}\quad\cdots①$$

$|\vec{a}|=\sqrt{a^2+\frac{a^2}{3}}=\frac{2}{\sqrt{3}}a$. $|\vec{b}|=\sqrt{b^2+\frac{b^2}{3}}=\frac{2}{\sqrt{3}}b$ より

$$\frac{|\vec{b}|}{|\vec{a}|}\vec{a}+\frac{|\vec{a}|}{|\vec{b}|}\vec{b}=\frac{b}{a}(a,\frac{a}{\sqrt{3}})+\frac{a}{b}(b,-\frac{b}{\sqrt{3}})=(a+b,-\frac{a}{\sqrt{3}}+\frac{b}{\sqrt{3}})\quad\cdots②$$

①②より題意は示された