京大理系 1975前期 ②

(i) 相加平均≧相乗平均より $\frac{x+1}{2} \geqq \sqrt{x}$

(ii) $\int_0^k \sqrt{x}\left(1-\frac{x}{k}\right)^k dx \leqq \int_0^k \frac{1}{2}(x+1)\left(1-\frac{x}{k}\right)^k dx = \frac{1}{2}\int_0^k x\left(1-\frac{x}{k}\right)^k dx + \frac{1}{2}\int_0^k \left(1-\frac{x}{k}\right)^k dx$

※ $\left\{\left(1-\frac{x}{k}\right)^{k+1}\right\}' = (k+1)\left(1-\frac{x}{k}\right)^k\left(-\frac{1}{k}\right)$ より $\left\{-\frac{k}{k+1}\left(1-\frac{x}{k}\right)^{k+1}\right\}' = \left(1-\frac{x}{k}\right)^k$

$= \frac{1}{2}\int_0^k x\left\{-\frac{k}{k+1}\left(1-\frac{x}{k}\right)^{k+1}\right\}' dx + \frac{1}{2}\left[-\frac{k}{k+1}\left(1-\frac{x}{k}\right)^{k+1}\right]_0^k$

$= \frac{1}{2}\left[-\frac{k}{k+1}x\left(1-\frac{x}{k}\right)^{k+1}\right]_0^k + \frac{1}{2}\frac{k}{k+1}\int_0^k \left(1-\frac{x}{k}\right)^{k+1} dx + \frac{1}{2}\frac{k}{k+1}$

※ $\left\{\left(1-\frac{x}{k}\right)^{k+2}\right\}' = (k+2)\left(1-\frac{x}{k}\right)^{k+1}\left(-\frac{1}{k}\right)$ より $\left\{-\frac{k}{k+2}\left(1-\frac{x}{k}\right)^{k+2}\right\}' = \left(1-\frac{x}{k}\right)^{k+1}$

$= \frac{1}{2}\frac{k}{k+1}\left[-\frac{k}{k+2}\left(1-\frac{x}{k}\right)^{k+2}\right]_0^k + \frac{1}{2}\frac{k}{k+1} = \frac{1}{2}\frac{k}{k+1}\frac{k}{k+2} + \frac{1}{2}\frac{k}{k+1} = \frac{1}{2}\frac{k}{k+1}\frac{k+k+2}{k+2} = \frac{1}{\cancel{2}}\frac{k}{\cancel{k+1}}\frac{\cancel{2(k+1)}}{k+2} < 1$