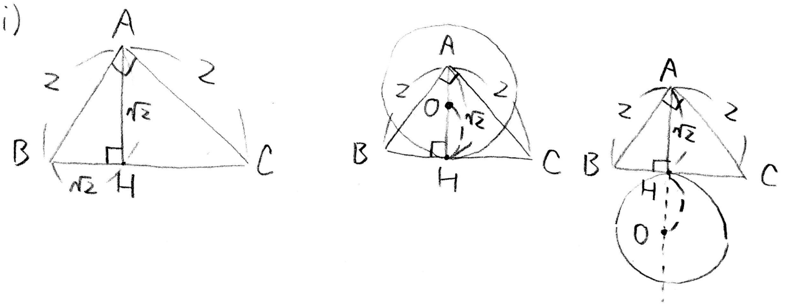
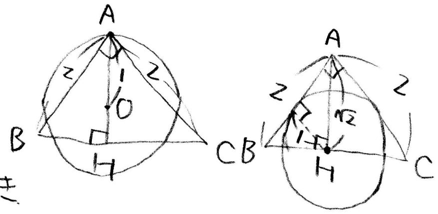
京大理系　1975前期　⑥

(i)



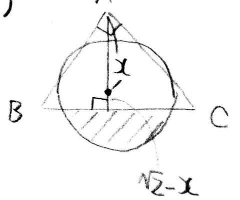
左図より、BCと2点で交わるのは
$\sqrt{2}-1<x<\sqrt{2}+1$ のとき　―①



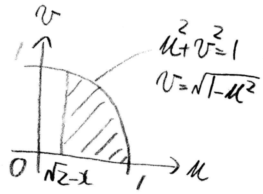
左図より、AB, ACと2点で交わるのは
$1<x<\sqrt{2}$ のとき　―②

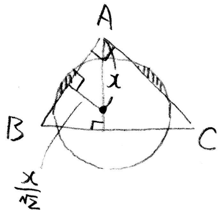
①②より $1<x<\sqrt{2}$ のとき

(ii)

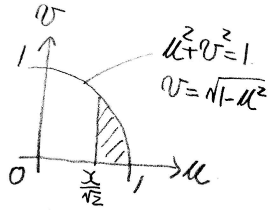


左図の斜線部の面積は
右図の斜線部の面積の2倍に等しく、
$2\int_{\sqrt{2}-x}^{1}\sqrt{1-u^2}\,du$　―③





左図の斜線部の面積は
右図の斜線部の面積の4倍に等しく、
$4\int_{\frac{x}{\sqrt{2}}}^{1}\sqrt{1-u^2}\,du$　―④



③④より $S=\pi-2\int_{\sqrt{2}-x}^{1}\sqrt{1-u^2}\,du-4\int_{\frac{x}{\sqrt{2}}}^{1}\sqrt{1-u^2}\,du=\pi-F(\sqrt{2}-x)-2F\left(\frac{x}{\sqrt{2}}\right)$

(iii) $\frac{dS}{dx}=-F'(\sqrt{2}-x)\cdot(-1)-2F'\left(\frac{x}{\sqrt{2}}\right)\cdot\frac{1}{\sqrt{2}}=-2\sqrt{1-(\sqrt{2}-x)^2}-\sqrt{2}\left(-2\sqrt{1-\frac{x^2}{2}}\right)$

※ $F(t)=-2\int_{1}^{t}\sqrt{1-u^2}\,du$ より $F'(t)=-2\sqrt{1-t^2}$

$=-2\sqrt{1-2+2\sqrt{2}x-x^2}+2\sqrt{2-x^2}=2\left(\sqrt{2-x^2}-\sqrt{-1+2\sqrt{2}x-x^2}\right)$

$\frac{dS}{dx}=0$ のとき　$2-\cancel{x^2}=-1+2\sqrt{2}x-\cancel{x^2}$　$x=\frac{3}{2\sqrt{2}}=\frac{3\sqrt{2}}{4}$

$\left.\frac{dS}{dx}\right|_{x=1}=2\left(\sqrt{2-1}-\sqrt{-1+2\sqrt{2}-1}\right)=2\left(1-\sqrt{2\sqrt{2}-2}\right)>0$

$\left.\frac{dS}{dx}\right|_{x=\sqrt{2}}=2\left(\sqrt{\cancel{2-2}}-\sqrt{-1+2\sqrt{2}\sqrt{2}-2}\right)=-2<0$

より、$x=\frac{3\sqrt{2}}{4}$ の前後で $\frac{dS}{dx}$ の符号は正から負に変わるから

Sは $x=\frac{3\sqrt{2}}{4}$ のとき最大値をとる