



P(θ)の座標を $(r \cos \theta, r \sin \theta), (r \cos \theta, -r \sin \theta)$ ($0 < \theta < \frac{\pi}{2}$) とおく

$$\vec{OP} = (r \cos \theta, r \sin \theta) \quad \vec{O'P} = (r \cos \theta - 1, r \sin \theta)$$

$$OP \perp O'P \Leftrightarrow \vec{OP} \cdot \vec{O'P} = 0, r^2 \cos^2 \theta - r^2 \cos \theta + r^2 \sin^2 \theta = 0, \cos \theta = 1$$

$$C' \text{の半径は } |\vec{OP}| = \sqrt{r^2 \cos^2 \theta - 2r^2 \cos \theta + 1 + r^2 \sin^2 \theta} = \sqrt{1 - r^2 \cos^2 \theta} = r \sin \theta$$

$$\begin{aligned} \text{左図の(1)の面積は } & \pi r^2 \cos^2 \frac{\theta}{2\pi} - r^2 \cos^2 \theta - r \cos \theta \cdot r \sin \theta \cdot \frac{1}{2} \\ & = \frac{1}{2} \theta r^2 \cos^2 \theta - \frac{1}{2} r^2 \sin^2 \theta \end{aligned}$$

$$\begin{aligned} \text{左図の(2)の面積は } & \pi r^2 \sin^2 \frac{\pi/2 - \theta}{2\pi} - (1 - r^2 \cos^2 \theta) r \cos \theta \cdot r \sin \theta \cdot \frac{1}{2} \\ & = \frac{1}{4} \pi r^2 \sin^2 \theta - \frac{1}{2} \theta r^2 \sin^2 \theta - \frac{1}{2} r^2 \sin^3 \theta \cos \theta \end{aligned}$$

求める面積は 上記の斜線部の面積であるから $2 \left(r \cos \theta \cdot r \sin \theta \cdot \frac{1}{2} - \frac{1}{2} \theta r^2 \cos^2 \theta + \frac{1}{2} r^2 \sin^2 \theta - \frac{1}{4} \pi r^2 \sin^2 \theta + \frac{1}{2} \theta r^2 \sin^2 \theta + \frac{1}{2} r^2 \sin^3 \theta \cos \theta \right)$

$$= r^2 \sin \theta \cos \theta - \theta r^2 \cos^2 \theta + r^2 \sin^2 \theta - \frac{1}{2} \pi r^2 \sin^2 \theta + \theta r^2 \sin^2 \theta + r^2 \sin^3 \theta \cos \theta$$

$$= 2 r^2 \sin \theta \cos \theta - \theta r^2 \cos^2 \theta - \frac{1}{2} \pi r^2 \sin^2 \theta + \theta r^2 \sin^2 \theta$$

$$f(\theta) = 2 r^2 \sin \theta \cos \theta - \theta r^2 \cos^2 \theta - \frac{1}{2} \pi r^2 \sin^2 \theta + \theta r^2 \sin^2 \theta \quad (0 < \theta < \frac{\pi}{2}) \text{ とおく}$$

$$\begin{aligned} f'(\theta) &= 2 r^2 \cos^2 \theta - 2 r^2 \sin \theta \cos \theta - r^2 \cos^2 \theta + \theta \cdot 2 r^2 \sin \theta \cos \theta - \pi r^2 \sin \theta \cos \theta + r^2 \sin^2 \theta + \theta \cdot 2 r^2 \sin \theta \cos \theta \\ &= r^2 \cos^2 \theta - r^2 \sin^2 \theta + (4\theta - \pi) r^2 \sin \theta \cos \theta \end{aligned}$$

$$g(\theta) = -r^2 \sin^2 \theta + r^2 \cos^2 \theta + (4\theta - \pi) r^2 \sin \theta \cos \theta \quad (0 < \theta < \frac{\pi}{2}) \text{ とおく}$$

$$\begin{aligned} g'(\theta) &= -2 r^2 \sin \theta \cos \theta - 2 r^2 \cos \theta \sin \theta + 4 r^2 \sin \theta \cos \theta + (4\theta - \pi) (r^2 \cos^2 \theta - r^2 \sin^2 \theta) \\ &= (4\theta - \pi) (r^2 \cos^2 \theta - r^2 \sin^2 \theta) \end{aligned}$$

$$g'(\theta) = 0 \text{ のとき, } 4\theta - \pi = 0 \text{ または } r^2 \cos^2 \theta - r^2 \sin^2 \theta = 0, \theta = \frac{\pi}{4}$$

θ	...	$\frac{\pi}{4}$...
$g(\theta)$	-	0	-
$g'(\theta)$	∨	0	∨

$g(\theta)$ の増減表は左表

よって $f(\theta)$ の増減表は右表

θ	...	$\frac{\pi}{4}$...
$f'(\theta)$	+	0	-
$f(\theta)$	↗	$1 - \frac{\pi}{4}$	↘

$$\ast f\left(\frac{\pi}{4}\right) = 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{\pi}{4} \cdot \frac{1}{2} - \frac{1}{2} \pi \cdot \frac{1}{2} + \frac{\pi}{4} \cdot \frac{1}{2} = 1 - \frac{\pi}{4}$$

ゆえに 求める値は $1 - \frac{\pi}{4}$