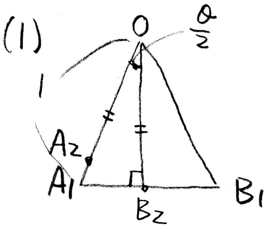
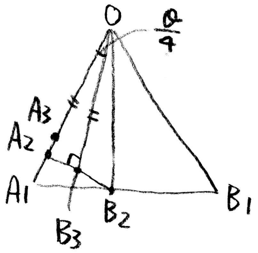
京大理系 1989前期 ①

(1)



左図より $\cos\frac{\theta}{2}=\frac{OB_2}{OA_1}=a_2$



左図より $\cos\frac{\theta}{4}=\frac{OB_3}{OA_2}=\frac{a_3}{a_2}$

$a_3\sin\frac{\theta}{4}=\cos\frac{\theta}{2}\cos\frac{\theta}{4}\sin\frac{\theta}{4}=\frac{1}{2}\cos\frac{\theta}{2}\sin\frac{\theta}{2}=\frac{1}{4}\sin\theta$

(2) $\cos\frac{\theta}{2^n}=\frac{OB_{n+1}}{OA_n}=\frac{a_{n+1}}{a_n}$

$b_n=a_n\sin\frac{\theta}{2^{n-1}}$ とする $\frac{b_{n+1}}{\sin\frac{\theta}{2^n}}=\frac{b_n}{\sin\frac{\theta}{2^{n-1}}}\cos\frac{\theta}{2^n}$, $b_{n+1}=\frac{b_n}{\cancel{\sin\frac{\theta}{2^{n-1}}}}\frac{1}{2}\cancel{\sin\frac{\theta}{2^{n-1}}}$

$b_n=\frac{1}{2}b_{n-1}=\left(\frac{1}{2}\right)^2b_{n-2}=\cdots=\left(\frac{1}{2}\right)^{n-1}b_1=\left(\frac{1}{2}\right)^{n-1}a_1\sin\theta=\left(\frac{1}{2}\right)^{n-1}\sin\theta$

$a_n=\frac{\left(\frac{1}{2}\right)^{n-1}\sin\theta}{\sin\frac{\theta}{2^{n-1}}}$

$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{1}{\frac{\sin\frac{\theta}{2^{n-1}}}{\frac{\theta}{2^{n-1}}}\theta}\sin\theta=\frac{\sin\theta}{\theta}$

$n\to\infty$ のとき $\frac{\theta}{2^{n-1}}\to 0$