京大理系　1994前期　②

(1)
$$
\begin{array}{ll}
a_0 = 1 & b_0 = 1 \\
a_1 = 2 & b_1 = 2 \\
a_2 = 2+1 = 3 & b_2 = 0 \\
a_3 = 3+2 = 5 & b_3 = 2 \\
a_4 = 5+3 = 8 & b_4 = 2 \\
a_5 = 8+5 = 13 & b_5 = 1 \\
a_6 = 13+8 = 21 & b_6 = 0 \\
a_7 = 21+13 = 34 & b_7 = 1 \\
a_8 = 34+21 = 55 & b_8 = 1 \\
a_9 = 55+34 = 89 & b_9 = 2
\end{array}
$$

(2) $b_8 = b_0$, $b_9 = b_1$，b_n, b_{n+1}が定まるとb_{n+2}が定まる，よってb_nは周期8の数列

ゆえに
$$
\begin{aligned}
C_{n+8} &= b_0 + b_1 + \cdots + b_n + b_{n+1} + b_{n+2} + b_{n+3} + b_{n+4} + b_{n+5} + b_{n+6} + b_{n+7} + b_{n+8} \\
&= C_n + b_0 + b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 \\
&= C_n + C_7
\end{aligned}
$$

(3)
$$
\left.
\begin{array}{ll}
C_0 = b_0 = 1 & 0+1 \leqq 1 \leqq \frac{3}{2}(0+1) \\
C_1 = C_0 + b_1 = 1+2 = 3 & 1+1 \leqq 3 \leqq \frac{3}{2}(1+1) \\
C_2 = C_1 + b_2 = 3+0 = 3 & 2+1 \leqq 3 \leqq \frac{3}{2}(2+1) \\
C_3 = C_2 + b_3 = 3+2 = 5 & 3+1 \leqq 5 \leqq \frac{3}{2}(3+1) \\
C_4 = C_3 + b_4 = 5+2 = 7 & 4+1 \leqq 7 \leqq \frac{3}{2}(4+1) \\
C_5 = C_4 + b_5 = 7+1 = 8 & 5+1 \leqq 8 \leqq \frac{3}{2}(5+1) \\
C_6 = C_5 + b_6 = 8+0 = 8 & 6+1 \leqq 8 \leqq \frac{3}{2}(6+1) \\
C_7 = C_6 + b_7 = 8+1 = 9 & 7+1 \leqq 9 \leqq \frac{3}{2}(7+1)
\end{array}
\right\} ①
$$

$$
\left.
\begin{array}{l}
k+1 \leqq C_k \leqq \frac{3}{2}(k+1) \text{ が成り立つとき} \\
C_{k+8} - (k+8+1) = C_k + \cancel{9} - k - \cancel{9} \geqq 1 \geqq 0. \\
\frac{3}{2}(k+8+1) - C_{k+8} = \frac{3}{2}(k+1) + 12 - C_k - 9 \geqq 3 \geqq 0 \\
\text{より}\ k+8+1 \leqq C_{k+8} \leqq \frac{3}{2}(k+8+1)
\end{array}
\right\} ②
$$

①②より数学的帰納法より題意は示された