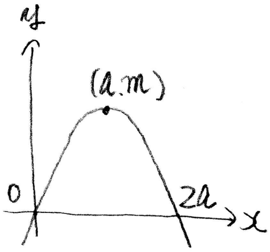
京大理系 1998前期 ④



放物線の方程式は、$y=kx(x-2a)$, $m=ka(-a)$, $k=-\frac{m}{a^2}$ より

$y=-\frac{m}{a^2}x(x-2a)$

$S_m=-\frac{2m}{a^2}\int_0^a(x^2-2ax)dx=-\frac{2m}{a^2}\left[\frac{x^3}{3}-\cancel{2}a\frac{x^2}{\cancel{2}}\right]_0^a=-2m\left(\frac{a}{3}-a\right)=\frac{4am}{3}$

$$2a+1+m+2\sum_{x=1}^{a-1}\left\{-\frac{m}{a^2}x(x-2a)-1\right\}<L_m\leqq 2a+1+m+2\sum_{x=1}^{a-1}\left\{-\frac{m}{a^2}x(x-2a)\right\}$$

$$\cancel{2a}+1+m-\frac{2m}{a^2}\left\{\frac{1}{6}(a-1)a(2a-1)-2a\frac{1}{\cancel{2}}(a-1)a\right\}-2(\cancel{a}-1)<L_m\leqq 2a+1+m-\frac{2m}{a^2}\left\{\frac{1}{6}(a-1)a(2a-1)-2a\frac{1}{\cancel{2}}(a-1)a\right\}$$

$$3+m-\frac{m}{3a}(2a^2-3a+1)+2m(a-1)<L_m\leqq 2a+1+m-\frac{m}{3a}(2a^2-3a+1)+2m(a-1)$$

$$3+\cancel{m}-\frac{2}{3}am+\cancel{m}-\frac{m}{3a}+2am-\cancel{2m}<L_m\leqq 2a+1+\cancel{m}-\frac{2}{3}ma+\cancel{m}-\frac{m}{3a}+2am-\cancel{2m}$$

$$\frac{4}{3}am-\frac{m}{3a}+3<L_m\leqq\frac{4}{3}am-\frac{m}{3a}+2a+1$$

$$\frac{3}{4am}\left(\frac{4}{3}am-\frac{m}{3a}+3\right)<\frac{L_m}{S_m}\leqq\frac{3}{4am}\left(\frac{4}{3}am-\frac{m}{3a}+2a+1\right)$$

$$1-\frac{1}{4a^2}+\frac{1}{4am}<\frac{L_m}{S_m}\leqq 1-\frac{1}{4a^2}+\frac{3}{2m}+\frac{3}{4am}$$

$\lim_{m\to\infty}\left(1-\frac{1}{4a^2}+\frac{1}{4am}\right)=1-\frac{1}{4a^2}$, $\lim_{m\to\infty}\left(1-\frac{1}{4a^2}+\frac{3}{2m}+\frac{3}{4am}\right)=1-\frac{1}{4a^2}$ より はさみうちの原理より $\lim_{m\to\infty}\frac{L_m}{S_m}=1-\frac{1}{4a^2}$