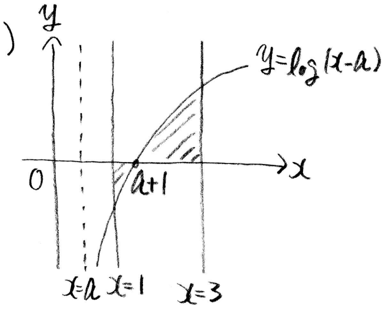
京大理系　1998前期　⑥

(1)



$V(a)=\int_1^3 \pi\{\log(x-a)\}^2dx=\int_{1-a}^{3-a}\pi(\log X)^2dX=\pi\int_{1-a}^{3-a}(X)'(\log X)^2dX$

$x-a=X$ とおく. $\begin{array}{c|c} x & 1\to 3 \\ \hline X & 1-a\to 3-a \end{array}$, $\dfrac{dX}{dx}=1$

$=\pi\left[X(\log X)^2\right]_{1-a}^{3-a}-\pi\int_{1-a}^{3-a}\cancel{X}\cdot 2\log X\cdot\dfrac{1}{\cancel{X}}dX$

$=\pi(3-a)\{\log(3-a)\}^2-\pi(1-a)\{\log(1-a)\}^2-2\pi\int_{1-a}^{3-a}(X)'\log X\,dX$

$=\pi(3-a)\{\log(3-a)\}^2-\pi(1-a)\{\log(1-a)\}^2-2\pi\left[X\log X\right]_{1-a}^{3-a}+2\pi\int_{1-a}^{3-a}\cancel{X}\dfrac{1}{\cancel{X}}dX$

$=\pi(3-a)\{\log(3-a)\}^2-\pi(1-a)\{\log(1-a)\}^2-2\pi(3-a)\log(3-a)+2\pi(1-a)\log(1-a)+4\pi$

(2) $V'(a)=-\pi\{\log(3-a)\}^2\cancel{+\pi(3-a)2\log(3-a)\cdot\dfrac{-1}{3-a}}+\pi\{\log(1-a)\}^2\cancel{-\pi(1-a)2\log(1-a)\cdot\dfrac{-1}{1-a}}$

$\cancel{+2\pi\log(3-a)}\cancel{-2\pi(3-a)\dfrac{-1}{3-a}}\cancel{-2\pi\log(1-a)}\cancel{+2\pi(1-a)\dfrac{-1}{1-a}}$

$=\pi\{\log(1-a)\}^2-\pi\{\log(3-a)\}^2$

$V'(a)=0$ のとき $\{\log(1-a)\}^2=\{\log(3-a)\}^2$

$\log(1-a)<0$, $\log(3-a)>0$ より $-\log(1-a)=\log(3-a)$. $\log\dfrac{1}{1-a}=\log(3-a)$

$\dfrac{1}{1-a}=3-a$. $(a-3)(a-1)=1$. $a^2-4a+2=0$. $a=2\pm\sqrt{4-2}=2\pm\sqrt{2}$. $0<a<1$ より $a=2-\sqrt{2}$

a	0	$\cdots$	$2-\sqrt{2}$	$\cdots$	1
$V'(a)$		$-$	0	$+$	
$V(a)$		$\searrow$	$V(2-\sqrt{2})$	$\nearrow$	

$V(a)$ の増減表は左表

$3-(2-\sqrt{2})=\sqrt{2}+1$, $1-(2-\sqrt{2})=\sqrt{2}-1=\dfrac{(\sqrt{2}-1)(\sqrt{2}+1)}{\sqrt{2}+1}=\dfrac{1}{\sqrt{2}+1}$ より

$V(2-\sqrt{2})=\pi(\sqrt{2}+1)\{\log(\sqrt{2}+1)\}^2-\pi(\sqrt{2}-1)\{\log(\sqrt{2}+1)\}^2-2\pi(\sqrt{2}+1)\log(\sqrt{2}+1)-2\pi(\sqrt{2}-1)\log(\sqrt{2}+1)+4\pi$

$=2\pi\{\log(\sqrt{2}+1)\}^2-4\sqrt{2}\pi\log(\sqrt{2}+1)+4\pi$

よって. $V(a)$ の最小値は $2\pi\{\log(\sqrt{2}+1)\}^2-4\sqrt{2}\pi\log(\sqrt{2}+1)+4\pi$