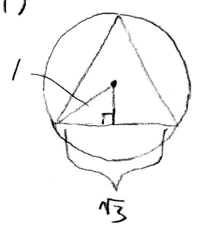
京大理系　1999前期 4

(1)



1. より $\alpha=x+yi+e^{i\theta}$, $\beta=x+yi+e^{i(\theta\pm\frac{2}{3}\pi)}$, $\gamma=x+yi+e^{i(\theta\mp\frac{2}{3}\pi)}$ とおける

$\alpha+\beta+\gamma=3x+3yi+e^{i\theta}(1+e^{\pm\frac{2}{3}\pi i}+e^{\mp\frac{2}{3}\pi i})$

$=3x+3yi+e^{i\theta}\{1+\cos(\pm\frac{2}{3}\pi)+i\sin(\pm\frac{2}{3}\pi)+\cos(\mp\frac{2}{3}\pi)+i\sin(\mp\frac{2}{3}\pi)\}$

$=3x+3yi+e^{i\theta}(1-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i-\frac{1}{2}\mp\frac{\sqrt{3}}{2}i)=3x+3yi$

2. より $x=1$, $y=0$. $\alpha=1+e^{i\theta}$, $\beta=1+e^{i(\theta\pm\frac{2}{3}\pi)}$, $\gamma=1+e^{i(\theta\mp\frac{2}{3}\pi)}$

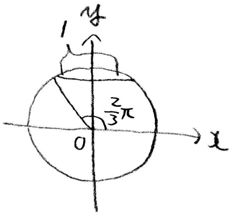
$z=e^{i\theta}$ より $\beta=1+ze^{\pm\frac{2}{3}\pi i}=1+z\{\cos(\pm\frac{2}{3}\pi)+i\sin(\pm\frac{2}{3}\pi)\}=1+z(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i)$

$\gamma=1+ze^{\mp\frac{2}{3}\pi i}=1+z\{\cos(\mp\frac{2}{3}\pi)+i\sin(\mp\frac{2}{3}\pi)\}=1+z(-\frac{1}{2}\mp\frac{\sqrt{3}}{2}i)$

(2) $\alpha\beta\gamma=(z+1)(1-\frac{1}{2}z\pm\frac{\sqrt{3}}{2}zi)(1-\frac{1}{2}z\mp\frac{\sqrt{3}}{2}zi)=(z+1)(1-z+\frac{1}{4}z^2+\frac{3}{4}z^2)=(z+1)(z^2-z+1)$

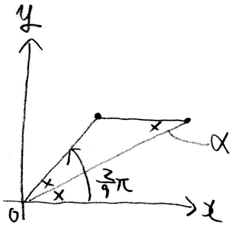
$=z^3\cancel{-z^2}\cancel{+z}\cancel{+z^2}\cancel{-z}+1$

3. より $|e^{i3\theta}+1|=1$

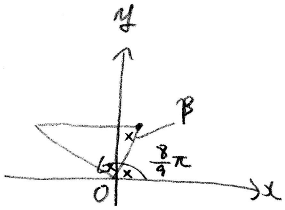


左図と3. より $3\theta=\frac{2}{3}\pi$　$\theta=\frac{2}{9}\pi$

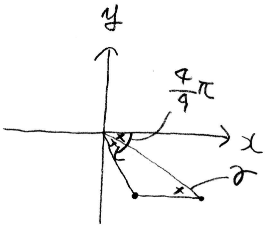
$\alpha=e^{\frac{2}{9}\pi i}+1$, $\beta=e^{\frac{8}{9}\pi i}+1$, $\gamma=e^{-\frac{4}{9}\pi i}+1=e^{\frac{14}{9}\pi i}+1$ とすると



$\arg\alpha=\frac{\pi}{9}$



$\arg\beta=\frac{4}{9}\pi$



$\arg\gamma=\frac{16}{9}\pi$

$0^\circ\leqq\arg\alpha\leqq\arg\beta\leqq\arg\gamma\leqq360^\circ$ より $\arg\alpha=20^\circ$, $\arg\beta=80^\circ$, $\arg\gamma=320^\circ$