京大理系　2002前期 ①

$$S_n = n(n-2)a_{n+1}$$
$$-)\ S_{n-1} = (n-1)(n-3)a_n$$

$$a_n = n(n-2)a_{n+1} - (n-1)(n-3)a_n$$

$$(n^2-4n+4)a_n = n(n-2)a_{n+1}$$

$$(n-2)^2 a_n = n(n-2)a_{n+1}$$

$n \geqq 3$のとき　$(n-2)a_n = na_{n+1}$　$a_{n+1} = \dfrac{n-2}{n}a_n$　, $n \geqq 4$のとき　$a_n = \dfrac{n-3}{n-1}a_{n-1}$

$a_4 = \dfrac{1}{3}a_3$ —①,

$n \geqq 5$のとき　$a_n = \dfrac{n-3}{n-1}a_{n-1} = \dfrac{n-3}{n-1}\dfrac{n-4}{n-2}a_{n-2} = \cdots = \dfrac{\cancel{n-3}}{n-1}\dfrac{\cancel{n-4}}{n-2}\cdots\dfrac{2}{\cancel{4}}\dfrac{1}{\cancel{3}}a_3 = \dfrac{2a_3}{(n-1)(n-2)}$ —②

①より②は $n=4$のときも成り立つ.

$-a_2 = S_1 = a_1 = 1$ より　$a_2 = -1$

$\dfrac{1}{n-2} - \dfrac{1}{n-1} = \dfrac{n-1-n+2}{(n-2)(n-1)} = \dfrac{1}{(n-1)(n-2)}$ より

$S_n = 2a_3\left(\cancel{\dfrac{1}{n-2}} - \dfrac{1}{n-1} + \cancel{\dfrac{1}{n-3}} - \cancel{\dfrac{1}{n-2}} + \cdots + \dfrac{1}{2} - \cancel{\dfrac{1}{3}}\right) + a_3 = 2a_3\left(-\dfrac{1}{n-1} + \dfrac{1}{2}\right) + a_3$

$\displaystyle\lim_{n\to\infty} S_n = a_3 + a_3 = 2a_3$

$2a_3 = 1$ より　$a_3 = \dfrac{1}{2}$

以上より.　$a_1 = 1$, $a_2 = -1$, $n \geqq 3$のとき　$a_n = \dfrac{1}{(n-1)(n-2)}$