京大理系　2003前期　①

(ii)より，$n \geqq 2$ のとき，$\log\frac{a_n}{a_{n-1}} = \log\frac{n-1}{n+1}$，$\frac{a_n}{a_{n-1}} = \frac{n-1}{n+1}$，$a_n = \frac{n-1}{n+1}a_{n-1}$

$a_2 = \frac{1}{3}a_1 = \frac{1}{3}$

$n \geqq 3$ のとき

$$a_n = \frac{n-1}{n+1}a_{n-1} = \frac{n-1}{n+1}\frac{n-2}{n}a_{n-2} = \cdots = \frac{\cancel{n-1}}{n+1}\frac{\cancel{n-2}}{n}\cdots\frac{2}{\cancel{4}}\frac{1}{\cancel{3}}a_1 = \frac{2}{(n+1)n}$$

これは $n=1, 2$ のときも成り立つ

$\frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$ より，$a_n = 2\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$$\sum_{k=1}^{n} a_k = 2\left(\cancel{\frac{1}{n}} - \frac{1}{n+1} + \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}} + \cdots + \frac{1}{1} - \cancel{\frac{1}{2}}\right) = 2\left(1 - \frac{1}{n+1}\right)$$