東工大　1994前期　③

(1) $\int_0^{\pi} e^{-x}\sin x\,dx=\int_0^{\pi}(-e^{-x})'\sin x\,dx=\left[-e^{-x}\sin x\right]_0^{\pi}+\int_0^{\pi}e^{-x}\cos x\,dx=\int_0^{\pi}(-e^{-x})'\cos x\,dx$

$=\left[-e^{-x}\cos x\right]_0^{\pi}+\int_0^{\pi}e^{-x}(-\sin x)\,dx=e^{-\pi}+1-\int_0^{\pi}e^{-x}\sin x\,dx$ より. $\int_0^{\pi}e^{-x}\sin x\,dx=\dfrac{e^{-\pi}+1}{2}$

(2) $\int_0^{n\pi}e^{-x}|\sin x|\,dx=\sum_{m=1}^{n}\int_{(m-1)\pi}^{m\pi}e^{-x}|\sin x|\,dx$ —①

$\int_{(m-1)\pi}^{m\pi}e^{-x}|\sin x|\,dx=\int_0^{\pi}e^{-X-(m-1)\pi}|\sin\{X+(m-1)\pi\}|\,dX=(e^{-\pi})^m e^{\pi}\int_0^{\pi}e^{-X}\sin X\,dX$

$\downarrow$

$x-(m-1)\pi=X$ とおく

| $x$ | $(m-1)\pi\to m\pi$ |
|---|---|
| $X$ | $0\to\pi$ |

$\dfrac{dX}{dx}=1$

$=(e^{-\pi})^m e^{\pi}\dfrac{e^{-\pi}+1}{2}$ より

① $=e^{\pi}\dfrac{e^{-\pi}+1}{2}\sum_{m=1}^{n}(e^{-\pi})^m=\cancel{e^{\pi}}\dfrac{e^{-\pi}+1}{2}\;\dfrac{\cancel{e^{-\pi}}\{1-(e^{-\pi})^n\}}{1-e^{-\pi}}=\dfrac{1}{2}\dfrac{e^{\pi}+1}{e^{\pi}-1}\{1-(e^{-\pi})^n\}$

よって. $\lim_{n\to\infty}\int_0^{n\pi}e^{-x}|\sin x|\,dx=\dfrac{1}{2}\dfrac{e^{\pi}+1}{e^{\pi}-1}$