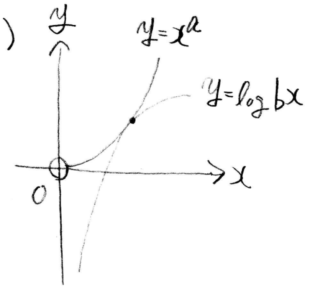
東工大 2008前期 ①

(1)



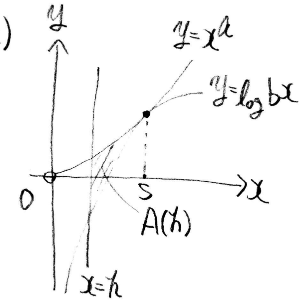
$f(x)=x^a,\ g(x)=\log bx$ とする $f'(x)=ax^{a-1}\quad g'(x)=\dfrac{1}{x}$

$x=s$ における $f(x), g(x)$ の接線の傾きは $as^{a-1},\ \dfrac{1}{s}$

$as^{a-1}=\dfrac{1}{s}$ より $s^a=\dfrac{1}{a}\quad s=\left(\dfrac{1}{a}\right)^{\frac{1}{a}},\quad t=\dfrac{1}{a}$

$\log\left\{b\left(\dfrac{1}{a}\right)^{\frac{1}{a}}\right\}=\dfrac{1}{a}$ より $\log\left\{b\left(\dfrac{1}{a}\right)^{\frac{1}{a}}\right\}=\log e^{\frac{1}{a}},\quad b\left(\dfrac{1}{a}\right)^{\frac{1}{a}}=e^{\frac{1}{a}}\quad b=(ea)^{\frac{1}{a}}$

(2)



$$A(h)=\int_h^s(x^a-\log bx)dx=\left[\frac{x^{a+1}}{a+1}\right]_h^s-\int_h^s(x)'\log bx\,dx$$

$$=\frac{s^{a+1}}{a+1}-\frac{h^{a+1}}{a+1}-\left[x\log bx\right]_h^s+\int_h^s x\frac{1}{x}dx=\frac{s^{a+1}}{a+1}-\frac{h^{a+1}}{a+1}-s\log bs+h\log bh+s-h$$

$$=\frac{1}{a+1}\left(\frac{1}{a}\right)^{1+\frac{1}{a}}-\frac{h^{a+1}}{a+1}-\left(\frac{1}{a}\right)^{\frac{1}{a}}\log\left\{(ea)^{\frac{1}{a}}\left(\frac{1}{a}\right)^{\frac{1}{a}}\right\}+h\log\left\{(ea)^{\frac{1}{a}}h\right\}+\left(\frac{1}{a}\right)^{\frac{1}{a}}-h$$

$$=\frac{1}{a^2+a}\left(\frac{1}{a}\right)^{\frac{1}{a}}-\frac{h^{a+1}}{a+1}-\frac{1}{a}\left(\frac{1}{a}\right)^{\frac{1}{a}}+h\log(ea)^{\frac{1}{a}}+h\log h+\left(\frac{1}{a}\right)^{\frac{1}{a}}-h$$

$$=\frac{1-a-1+a^2+a}{a^2+a}\left(\frac{1}{a}\right)^{\frac{1}{a}}-\frac{h^{a+1}}{a+1}+h\log(ea)^{\frac{1}{a}}+h\log h-h$$

$$=\frac{a}{a+1}\left(\frac{1}{a}\right)^{\frac{1}{a}}-\frac{h^{a+1}}{a+1}+h\log(ea)^{\frac{1}{a}}+h\log h-h \quad ―①$$

ここで $\displaystyle\lim_{h\to 0}h\log h=\lim_{k\to\infty}\frac{1}{k}\log\frac{1}{k}=\lim_{k\to\infty}\left(-\frac{\log k}{k}\right)$ ―②

$\dfrac{1}{h}=k$ とおく.
$h\to 0$ のとき $k\to\infty$

$h(k)=\sqrt{k}-\log k\ (k>0)$ とする $h'(k)=\dfrac{1}{2}\dfrac{1}{\sqrt{k}}-\dfrac{1}{k}=\dfrac{1}{\sqrt{k}}\left(\dfrac{1}{2}-\dfrac{1}{\sqrt{k}}\right).\quad h'(k)=0$ のとき $k=4$

k	$\cdots$	4	$\cdots$
$h'(k)$	$-$	0	$+$
$h(k)$	↘	$2-\log 4$	↗

$h(k)$ の増減表は左表

$\log 4<\log e^2=2$ より $2-\log 4>0.\quad \sqrt{k}>\log k$

k が十分大きいとき $-\dfrac{\sqrt{k}}{k}<-\dfrac{\log k}{k}<0,\quad -\dfrac{1}{\sqrt{k}}<-\dfrac{\log k}{k}<0$

$\displaystyle\lim_{k\to\infty}\left(-\frac{1}{\sqrt{k}}\right)=0$ より. はさみうちの原理より $\displaystyle\lim_{k\to\infty}\left(-\frac{\log k}{k}\right)=0$ ②より $\displaystyle\lim_{h\to 0}h\log h=0$ ―③

①③より $\displaystyle\lim_{h\to 0}A(h)=\frac{a}{a+1}\left(\frac{1}{a}\right)^{\frac{1}{a}}$