東工大　2010前期 ①

(1) $f'(x)=\sin x-\sin x-x\cos x=-x\cos x$　$f'(x)=0$ のとき $x=0,\ \frac{\pi}{2}+n$ (nは任意の整数)

x	0	$\cdots$	$\frac{\pi}{2}$	$\cdots$	π
$f'(x)$	0	$-$	0	$+$	$+$
$f(x)$	0	$\searrow$	$1-\frac{\pi}{2}$	$\nearrow$	2

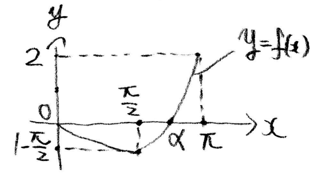
$0\leqq x\leqq\pi$ のとき

$f(x)$の増減表は左表

$f(x)$のグラフは右図

よって $0<x<\pi$ において

$f(x)=0$ は唯一の解を持つ



(2) $J=\int_0^{\alpha}(-1+\cos x+x\sin x)dx+\int_{\alpha}^{\pi}(1-\cos x-x\sin x)dx=\Big[-x+\sin x\Big]_0^{\alpha}+\int_0^{\alpha}x(-\cos x)'dx+\Big[x-\sin x\Big]_{\alpha}^{\pi}+\int_{\alpha}^{\pi}x(\cos x)'dx$

$=-\alpha+\sin\alpha+\Big[-x\cos x\Big]_0^{\alpha}+\int_0^{\alpha}\cos x\,dx+\pi-\alpha+\sin\alpha+\Big[x\cos x\Big]_{\alpha}^{\pi}-\int_{\alpha}^{\pi}\cos x\,dx$

$=-\alpha+\sin\alpha-\alpha\cos\alpha+\Big[\sin x\Big]_0^{\alpha}+\cancel{\pi}-\alpha+\sin\alpha-\cancel{\pi}-\alpha\cos\alpha-\Big[\sin x\Big]_{\alpha}^{\pi}$

$=-2\alpha+2\sin\alpha-2\alpha\cos\alpha+\sin\alpha+\sin\alpha=-2\alpha+4\sin\alpha-2\alpha\cos\alpha$

ここで $\sin^2\alpha+\cos^2\alpha=1$ より $\cos^2\alpha=1-\sin^2\alpha$. (1)より $\frac{\pi}{2}<\alpha<\pi$ より $\cos\alpha<0$ より $\cos\alpha=-\sqrt{1-\sin^2\alpha}$

$1-\cos\alpha-\alpha\sin\alpha=0$ より $1+\sqrt{1-\sin^2\alpha}-\alpha\sin\alpha=0$.　$\alpha=\frac{1+\sqrt{1-\sin^2\alpha}}{\sin\alpha}$

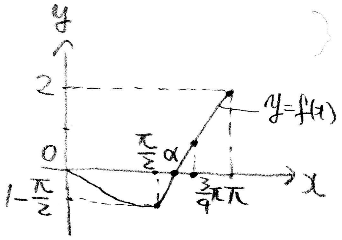
よって $J=-2\frac{1+\sqrt{1-\sin^2\alpha}}{\sin\alpha}+4\sin\alpha+2\frac{1+\sqrt{1-\sin^2\alpha}}{\sin\alpha}\sqrt{1-\sin^2\alpha}=\frac{\cancel{-2}\cancel{-2\sqrt{1-\sin^2\alpha}}+4\sin^2\alpha+\cancel{2\sqrt{1-\sin^2\alpha}}+\cancel{2}-2\sin^2\alpha}{\sin\alpha}$

$=\frac{2\sin^2\alpha}{\sin\alpha}=2\sin\alpha$

(3) $f\left(\frac{3}{4}\pi\right)=1+\frac{1}{\sqrt{2}}-\frac{3}{4}\pi\frac{1}{\sqrt{2}}=\frac{4\sqrt{2}+4-3\pi}{4\sqrt{2}}=\frac{\sqrt{2}+1-\frac{3}{4}\pi}{\sqrt{2}}$

$$\begin{array}{r} 0.75 \\ \times\ \ 3.2 \\ \hline 150 \\ 225\ \ \\ \hline 2.400 \end{array}$$

$\sqrt{2}+1-\frac{3}{4}\pi>1.4+1-0.75\times3.2=0$ より $f\left(\frac{3}{4}\pi\right)>0$



よって左図より $\frac{\pi}{2}<\alpha<\frac{3}{4}\pi$

右図より $\sin\alpha>\frac{1}{\sqrt{2}}$

$2\sin\alpha>\sqrt{2}$,　$J>\sqrt{2}$