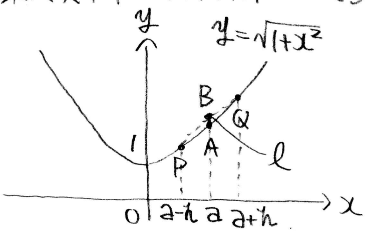
東大文科　1961前期　⑤



直線PQの方程式は

$$y-\sqrt{1+(a-h)^2}=\frac{\sqrt{1+(a+h)^2}-\sqrt{1+(a-h)^2}}{2h}\{x-(a-h)\}$$

Bのy座標は

$$\frac{\sqrt{1+(a+h)^2}-\sqrt{1+(a-h)^2}}{2\cancel{h}}\cancel{h}+\sqrt{1+(a-h)^2}=\frac{1}{2}\sqrt{1+(a+h)^2}+\frac{1}{2}\sqrt{1+(a-h)^2}$$

$$l=\frac{1}{2}\sqrt{1+(a+h)^2}+\frac{1}{2}\sqrt{1+(a-h)^2}-\sqrt{1+a^2}=\frac{1}{2}\sqrt{1+(a+h)^2}-\frac{1}{2}\sqrt{1+a^2}-\frac{1}{2}\sqrt{1+a^2}+\frac{1}{2}\sqrt{1+(a-h)^2}$$

$$=\frac{1}{2}\frac{\{\sqrt{1+(a+h)^2}-\sqrt{1+a^2}\}\{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}\}}{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}}-\frac{1}{2}\frac{\{\sqrt{1+a^2}-\sqrt{1+(a-h)^2}\}\{\sqrt{1+a^2}+\sqrt{1+(a-h)^2}\}}{\sqrt{1+a^2}+\sqrt{1+(a-h)^2}}$$

$$=\frac{1}{2}\frac{\cancel{1}+\cancel{a^2}+2ah+h^2-\cancel{1}-\cancel{a^2}}{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}}-\frac{1}{2}\frac{\cancel{1}+\cancel{a^2}-\cancel{1}-\cancel{a^2}+2ah-h^2}{\sqrt{1+a^2}+\sqrt{1+(a-h)^2}}=\frac{1}{2}\frac{h^2+2ah}{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}}+\frac{1}{2}\frac{h^2-2ah}{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}}$$

$$\frac{l}{h^2}=\frac{1}{2}\frac{1}{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}}+\frac{1}{2}\frac{1}{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}}+\frac{a}{h}\underline{\left\{\frac{1}{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}}-\frac{1}{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}}\right\}}$$

〜〜部 $=\dfrac{\sqrt{1+(a-h)^2}+\cancel{\sqrt{1+a^2}}-\sqrt{1+(a+h)^2}-\cancel{\sqrt{1+a^2}}}{\{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}\}}=\dfrac{\{\sqrt{1+(a-h)^2}-\sqrt{1+(a+h)^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+(a+h)^2}\}}{\{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+(a+h)^2}\}}$

$=\dfrac{\cancel{1}+\cancel{a^2}-2ah+\cancel{h^2}-\cancel{1}-\cancel{a^2}-2ah-\cancel{h^2}}{\{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+(a+h)^2}\}}$ より

$$\frac{l}{h^2}=\frac{1}{2}\frac{1}{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}}+\frac{1}{2}\frac{1}{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}}-\frac{4a^2}{\{\sqrt{1+(a+h)^2}+\sqrt{1+a^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+a^2}\}\{\sqrt{1+(a-h)^2}+\sqrt{1+(a+h)^2}\}}$$

$$\lim_{h\to0}\frac{l}{h^2}=\frac{1}{2}\frac{1}{\sqrt{1+a^2}+\sqrt{1+a^2}}+\frac{1}{2}\frac{1}{\sqrt{1+a^2}+\sqrt{1+a^2}}-\frac{4a^2}{(\sqrt{1+a^2}+\sqrt{1+a^2})(\sqrt{1+a^2}+\sqrt{1+a^2})(\sqrt{1+a^2}+\sqrt{1+a^2})}$$

$$=\frac{1}{2\sqrt{1+a^2}}-\frac{\cancel{4}a^2}{\underset{2}{\cancel{8}}(1+a^2)\sqrt{1+a^2}}=\frac{1+\cancel{a^2}-\cancel{a^2}}{2(1+a^2)\sqrt{1+a^2}}=\frac{1}{2(1+a^2)\sqrt{1+a^2}}$$