東大理科　1961前期　⑥

(i) $f'(x)=a\cos x-b\sin x+c\cos 2x\cdot 2$

$f'(\frac{\pi}{4})=0$ より $\frac{a}{\sqrt{2}}-\frac{b}{\sqrt{2}}=0$. $a=b$

$f(\frac{\pi}{4})=6\sqrt{2}$ より $\frac{a}{\sqrt{2}}+\frac{a}{\sqrt{2}}+c=6\sqrt{2}$. $c=-\sqrt{2}a+6\sqrt{2}$

$\int_0^{2\pi}f(x)\cos x\,dx=\int_0^{2\pi}\{a\sin x+a\cos x+(-\sqrt{2}a+6\sqrt{2})\sin 2x\}\cos x\,dx$

$=\int_0^{2\pi}\{\frac{a}{2}\sin 2x+a\frac{1+\cos 2x}{2}+(-\sqrt{2}a+6\sqrt{2})\frac{1}{2}\sin x(1-\sin^2 x)\}dx$

$=\frac{a}{2}[x]_0^{2\pi}=\pi a$ であるから　※ $\int_0^{2\pi}\sin 2x\,dx=\int_0^{2\pi}\cos 2x\,dx=\int_0^{2\pi}\sin x\,dx=\int_0^{2\pi}\sin^3 x\,dx=0$

$\pi a=5\pi$. $a=5$.

よって $b=5$, $c=\sqrt{2}$

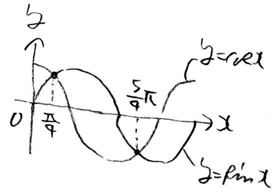
(ii) $f(x)=5\sin x+5\cos x+\sqrt{2}\sin 2x$

$f'(x)=5\cos x-5\sin x+\sqrt{2}\cos 2x\cdot 2$

$=5(\cos x-\sin x)+2\sqrt{2}(\cos x-\sin x)(\cos x+\sin x)$

$=2\sqrt{2}(\cos x-\sin x)(\cos x+\sin x+\frac{5}{4}\sqrt{2})$

ここで $\sin(x+\frac{\pi}{4})=\sin x\frac{1}{\sqrt{2}}+\cos x\frac{1}{\sqrt{2}}$, $\sin x+\cos x=\sqrt{2}\sin(x+\frac{\pi}{4})$ であるから



$f'(x)=4(\cos x-\sin x)\{\sin(x+\frac{\pi}{4})+\frac{5}{4}\}$

$\sin x=\cos x$, $x=\frac{\pi}{4}$, $\frac{5}{4}\pi$ のとき $f'(x)=0$

x	0	$\cdots$	$\frac{\pi}{4}$	$\cdots$	$\frac{5}{4}\pi$	$\cdots$	2π
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$	5	↗	$6\sqrt{2}$	↘	$-4\sqrt{2}$	↗	5

$f(x)$の増減表は左表のようになる

よって $f(x)$ は $x=\frac{5}{4}\pi$ で最小値 $-4\sqrt{2}$ をとる

$f(\frac{5}{4}\pi)=-\frac{5}{\sqrt{2}}-\frac{5}{\sqrt{2}}+\sqrt{2}=-5\sqrt{2}+\sqrt{2}=-4\sqrt{2}$