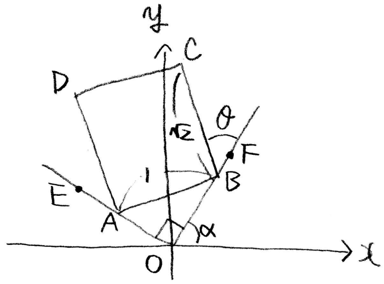
東大 文科 1962 前期 ④



(i) Cの座標を(C_x, C_y)とすると

$C_x = OB\cos\alpha + \sqrt{2}\cos(\alpha+\theta)$ —①

$C_y = OB\sin\alpha + \sqrt{2}\sin(\alpha+\theta)$ —②

$\angle OAB = \theta$ より $OB = \sin\theta$

$\tan\alpha = \sqrt{2}$ より $\dfrac{1-\cos^2\alpha}{\cos^2\alpha} = 2$, $\dfrac{1}{\cos^2\alpha} = 3$ $0<\alpha<90°$より $\cos\alpha>0$, $\cos\alpha = \dfrac{\sqrt{3}}{3}$

$\sin^2\alpha + \dfrac{1}{3} = 1$ $\sin\alpha = \dfrac{\sqrt{6}}{3}$

①より $C_x = \sin\theta\dfrac{\sqrt{3}}{3} + \sqrt{2}\cos\alpha\cos\theta - \sqrt{2}\sin\alpha\sin\theta = \dfrac{\sqrt{3}}{3}\sin\theta + \dfrac{\sqrt{6}}{3}\cos\theta - \dfrac{2\sqrt{3}}{3}\sin\theta = -\dfrac{\sqrt{3}}{3}\sin\theta + \dfrac{\sqrt{6}}{3}\cos\theta$

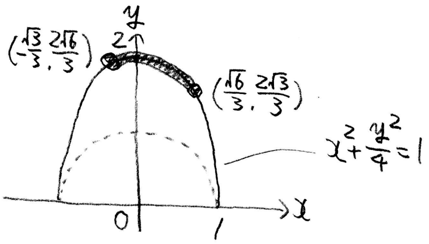
②より $C_y = \sin\theta\dfrac{\sqrt{6}}{3} + \sqrt{2}\sin\alpha\cos\theta + \sqrt{2}\cos\alpha\sin\theta = \dfrac{\sqrt{6}}{3}\sin\theta + \dfrac{2\sqrt{3}}{3}\cos\theta + \dfrac{\sqrt{6}}{3}\sin\theta = \dfrac{2\sqrt{6}}{3}\sin\theta + \dfrac{2\sqrt{3}}{3}\cos\theta$

(ii) $0°<\theta<90°$

$\theta=0°$のとき $C_x = \dfrac{\sqrt{6}}{3}$, $C_y = \dfrac{2\sqrt{3}}{3}$, $\theta = 90°$のとき $C_x = -\dfrac{\sqrt{3}}{3}$, $C_y = \dfrac{2\sqrt{6}}{3}$

$$\underbrace{\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\frac{\sqrt{6}}{3}\\ \frac{\sqrt{3}}{3}\end{pmatrix}} = \begin{pmatrix}-\frac{\sqrt{3}}{3}\sin\theta + \frac{\sqrt{6}}{3}\cos\theta\\ \frac{\sqrt{6}}{3}\sin\theta + \frac{\sqrt{3}}{3}\cos\theta\end{pmatrix} = \begin{pmatrix}C_x\\ \frac{1}{2}C_y\end{pmatrix}$$

ここで $\sqrt{\left(\frac{\sqrt{6}}{3}\right)^2 + \left(\frac{\sqrt{3}}{3}\right)^2} = \sqrt{\frac{6}{9}+\frac{3}{9}} = 1$ より 〰部は $0°<\theta<90°$のとき 中心$(0,0)$ 半径1の円の一部を表す.



よって 求める軌跡は左図の太線部