東大文科 1963前期 ③

(1) $g(x)=\dfrac{4x-2}{5-x}-x=\dfrac{4x-2-x(5-x)}{5-x}=\dfrac{-x^2+x+2}{x-5}$ $(x\neq5)$ とする

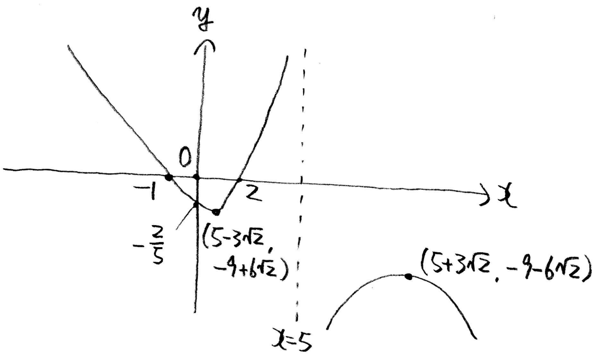
$g'(x)=\dfrac{(-2x+1)(x-5)-(-x^2+x+2)}{(x-5)^2}=\dfrac{-2x^2+10x+x-5+x^2-x-2}{(x-5)^2}=\dfrac{-x^2+10x-7}{(x-5)^2}$

$g'(x)=0$のとき. $x^2-10x+7=0$. $x=5\pm\sqrt{25-7}=5\pm3\sqrt{2}$

$g(x)$の増減表は下表

x	$-\infty$	$\cdots$	$5-3\sqrt{2}$	$\cdots$	5	$\cdots$	$5+3\sqrt{2}$	$\cdots$	∞
$g'(x)$		$-$	0	$+$		$+$	0	$-$	
$g(x)$	∞	$\searrow$	$-9+6\sqrt{2}$	$\nearrow$	$\infty \mid -\infty$	$\nearrow$	$-9-6\sqrt{2}$	$\searrow$	$-\infty$

yのグラフは下図



$\lim_{x\to-\infty} g(x)=\lim_{x\to-\infty}\dfrac{-x+1+\frac{2}{x}}{1-\frac{5}{x}}=\infty$

$-5^2+5+2=-18$

$\lim_{x\to5-0} g(x)=\infty$. $\lim_{x\to5+0} g(x)=-\infty$

$\lim_{x\to\infty} g(x)=\lim_{x\to\infty}\dfrac{-x+1+\frac{2}{x}}{1-\frac{5}{x}}=-\infty$

$g(5-3\sqrt{2})=\dfrac{20-12\sqrt{2}-2}{5-5+3\sqrt{2}}-5+3\sqrt{2}$

$=\dfrac{18\sqrt{2}-24}{6}-5+3\sqrt{2}=3\sqrt{2}-4-5+3\sqrt{2}=-9+6\sqrt{2}$

$g(5+3\sqrt{2})=\dfrac{20+12\sqrt{2}-2}{5-5-3\sqrt{2}}-5-3\sqrt{2}$

$=\dfrac{18\sqrt{2}+24}{-6}-5-3\sqrt{2}=-3\sqrt{2}-4-5-3\sqrt{2}=-9-6\sqrt{2}$

$x^2-x-2=0$. $x=\dfrac{1\pm\sqrt{1+8}}{2}=\dfrac{1\pm3}{2}=-1, 2$.

(2) (1)より $f(x)-x\leqq-9-6\sqrt{2}$, $f(x)-x\geqq-9+6\sqrt{2}$

(3) $f(x)>x$ のとき $f(x)-x>0$.

(1)より $x<-1$, $2<x<5$