東大理科　1966前期　③

nが3で割り切れる数のとき $\overrightarrow{P_nP_{n+1}} = 2^n\begin{pmatrix}1\\0\end{pmatrix}$

nが3で割ると1余る数のとき $\overrightarrow{P_nP_{n+1}} = 2^n\begin{pmatrix}-\frac{1}{2}\\\frac{\sqrt{3}}{2}\end{pmatrix}$

nが3で割ると2余る数のとき $\overrightarrow{P_nP_{n+1}} = 2^n\begin{pmatrix}-\frac{1}{2}\\-\frac{\sqrt{3}}{2}\end{pmatrix}$

$(1,0)=\vec{a}$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)=\vec{b}$, $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)=\vec{c}$ とすると、

$$\overrightarrow{P_0P_{3n}} = \overrightarrow{P_0P_1} + \overrightarrow{P_1P_2} + \cdots + \overrightarrow{P_{3n-1}P_{3n}}$$

$$= \left(\overrightarrow{P_0P_1} + \overrightarrow{P_3P_4} + \cdots + \overrightarrow{P_{3n-3}P_{3n-2}}\right) + \left(\overrightarrow{P_1P_2} + \overrightarrow{P_4P_5} + \cdots + \overrightarrow{P_{3n-2}P_{3n-1}}\right) + \left(\overrightarrow{P_2P_3} + \overrightarrow{P_5P_6} + \cdots + \overrightarrow{P_{3n-1}P_{3n}}\right)$$

$$= \left(2^0 + 2^3 + \cdots + 2^{3n-3}\right)\vec{a} + \left(2^1 + 2^4 + \cdots + 2^{3n-2}\right)\vec{b} + \left(2^2 + 2^5 + \cdots + 2^{3n-1}\right)\vec{c}$$

$$= \frac{8^n-1}{8-1}\vec{a} + 2\frac{8^n-1}{8-1}\vec{b} + 4\frac{8^n-1}{8-1}\vec{c} = \frac{8^n-1}{7}\left(\vec{a}+2\vec{b}+4\vec{c}\right)$$

$$= \frac{8^n-1}{7}\left(\cancel{1}-\cancel{1}-2, \sqrt{3}-2\sqrt{3}\right) = \frac{8^n-1}{7}\left(-2, -\sqrt{3}\right)$$

よって P_{3n} の座標は $\left(-\frac{2}{7}(8^n-1), -\frac{\sqrt{3}}{7}(8^n-1)\right)$