東大理科　1971前期　③

$$\int_0^1 \{f(x)^2 + a^2x^2 + b^2 - 2axf(x) - 2bf(x) + 2abx\}dx = \int_0^1 f(x)^2dx + a^2\left[\frac{x^3}{3}\right]_0^1 + b^2\left[x\right]_0^1 - 2a\cdot 3 - 2b\cdot 2 + 2ab\left[\frac{x^2}{2}\right]_0^1$$

$$= \int_0^1 f(x)^2dx + \frac{1}{3}a^2 + b^2 - 6a - 4b + ab = \int_0^1 f(x)^2dx + \frac{1}{3}a^2 + (b-6)a + b^2 - 4b$$

$$= \int_0^1 f(x)^2dx + \frac{1}{3}\left\{a^2 + 3(b-6)a + \frac{9}{4}(b-6)^2\right\} - \frac{3}{4}(b-6)^2 + b^2 - 4b = \int_0^1 f(x)^2dx + \frac{1}{3}\left\{a + \frac{3}{2}(b-6)\right\}^2 - \frac{3}{4}b^2 + 9b - 27 + b^2 - 4b$$

$$= \int_0^1 f(x)^2dx + \frac{1}{3}\left\{a + \frac{3}{2}(b-6)\right\}^2 + \frac{1}{4}b^2 + 5b - 27 = \int_0^1 f(x)^2dx + \frac{1}{3}\left\{a + \frac{3}{2}(b-6)\right\}^2 + \frac{1}{4}(b^2 + 20b + 100) - 25 - 27$$

$$= \int_0^1 f(x)^2dx + \frac{1}{3}\left\{a + \frac{3}{2}(b-6)\right\}^2 + \frac{1}{4}(b+10)^2 - 52.$$

よって $b=-10$, $a-24=0$. $a=24$