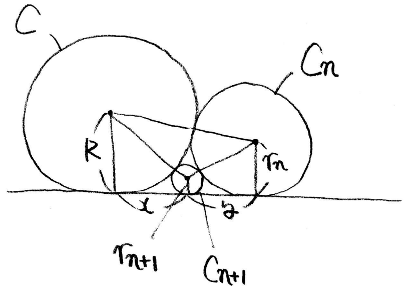
東大理科　1972前期　③



左図より
$$\begin{cases} (x+y)^2+(R-r_n)^2=(R+r_n)^2, & (x+y)^2=4Rr_n \\ x^2+(R-r_{n+1})^2=(R+r_{n+1})^2, & x^2=4Rr_{n+1} \\ y^2+(r_n-r_{n+1})^2=(r_n+r_{n+1})^2, & y^2=4r_nr_{n+1} \end{cases} \quad ——①$$

①より.
$$\cancel{4}Rr_{n+1}+\cancel{2}\cdot 2\sqrt{R}\sqrt{r_{n+1}}\cdot 2\sqrt{r_n}\sqrt{r_{n+1}}+\cancel{4}r_nr_{n+1}=\cancel{4}Rr_n$$
$$(R+2\sqrt{R}\sqrt{r_n}+r_n)r_{n+1}=Rr_n$$
$$\frac{1}{r_n}+\frac{2}{\sqrt{R}\sqrt{r_n}}+\frac{1}{R}=\frac{1}{r_{n+1}},\quad \left(\frac{1}{\sqrt{r_n}}+\frac{1}{\sqrt{R}}\right)^2=\frac{1}{r_{n+1}},\quad \frac{1}{\sqrt{r_n}}+\frac{1}{\sqrt{R}}=\frac{1}{\sqrt{r_{n+1}}} \ —②$$

$S_n=\dfrac{1}{\sqrt{r_n}}$ とおく.

$$\begin{aligned} S_n&=S_{n-1}+\frac{1}{\sqrt{R}} \\ S_{n-1}&=S_{n-2}+\frac{1}{\sqrt{R}} \\ &\ \vdots \\ +)\ S_2&=S_1+\frac{1}{\sqrt{R}} \\ \hline S_n&=S_1+(n-1)\frac{1}{\sqrt{R}} \end{aligned}$$

②より. $\dfrac{1}{\sqrt{r_1}}=\dfrac{1}{\sqrt{R}}+\dfrac{1}{\sqrt{R'}}$ より. $S_1=\dfrac{1}{\sqrt{R}}+\dfrac{1}{\sqrt{R'}}$

$$S_n=\frac{n}{\sqrt{R}}+\frac{1}{\sqrt{R'}}$$

$$r_n=\frac{1}{\left(\frac{n}{\sqrt{R}}+\frac{1}{\sqrt{R'}}\right)^2}=\frac{1}{\frac{n^2}{R}+\frac{2n}{\sqrt{RR'}}+\frac{1}{R'}}$$

$$\lim_{n\to\infty}n^2r_n=\lim_{n\to\infty}\frac{1}{\frac{1}{R}+\frac{2}{n\sqrt{RR'}}+\frac{1}{n^2R'}}=R$$