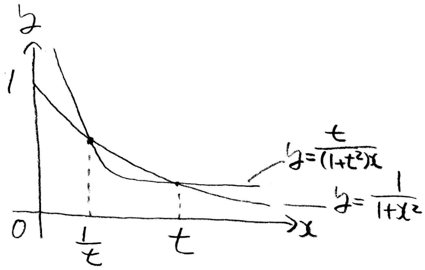
東大理科　1973前期　⑤



$y=\frac{t}{(1+t^2)x}$ と $y=\frac{1}{1+x^2}$ の交点の x 座標は．

$\frac{t}{(1+t^2)x}=\frac{1}{1+x^2}$．$tx^2-(t^2+1)x+t=0$

$x=\frac{t^2+1\pm\sqrt{t^4+2t^2+1-4t^2}}{2t}=\frac{t^2+1\pm\sqrt{(t^2-1)^2}}{2t}=\frac{t^2+1\pm(t^2-1)}{2t}=\frac{2t^2}{2t},\frac{2}{2t}=t,\frac{1}{t}$

より．$\frac{1}{t}$，t

$f(t)=\int_{\frac{1}{t}}^{t}\left\{\frac{1}{1+x^2}-\frac{t}{(1+t^2)x}\right\}dx=\int_{\frac{1}{t}}^{t}\frac{1}{1+x^2}dx-\frac{t}{1+t^2}\int_{\frac{1}{t}}^{t}\frac{1}{x}dx=\int_{\frac{1}{t}}^{t}\frac{1}{1+x^2}dx-2\frac{t\log t}{1+t^2}$

※ $\int_{\frac{1}{t}}^{t}\frac{1}{x}dx=\left[\log x\right]_{\frac{1}{t}}^{t}=\log t-\log\frac{1}{t}=2\log t$

$F(x)$ を $F'(x)=\frac{1}{1+x^2}$ を満たす関数とすると．

$f(t)=F(t)-F\left(\frac{1}{t}\right)-2\frac{t\log t}{1+t^2}$

$f'(t)=F'(t)-F'\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)-2\frac{\left(\log t+t\frac{1}{t}\right)(1+t^2)-t\log t\cdot 2t}{(1+t^2)^2}=\frac{1}{1+t^2}+\frac{1}{t^2}\frac{1}{1+\frac{1}{t^2}}-2\frac{\log t+t^2\log t+1+t^2-2t^2\log t}{(1+t^2)^2}$

$=\frac{2}{1+t^2}-2\frac{1+t^2+(1-t^2)\log t}{(1+t^2)^2}=2\frac{t^2-1}{(t^2+1)^2}\log t$