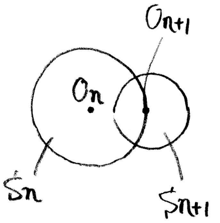
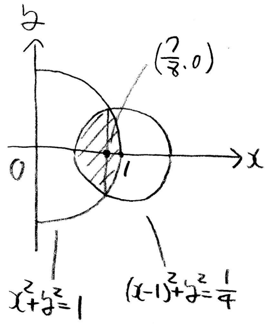
東大理科　1975前期　⑤

(i)





$x^2+y^2=1$ と $(x-1)^2+y^2=\frac{1}{4}$ の交点のx座標は

$(x-1)^2-x^2+1=\frac{1}{4}$, $\cancel{x^2}-2x+1-\cancel{x^2}+1=\frac{1}{4}$, $2x=\frac{7}{4}$, $x=\frac{7}{8}$ より $\frac{7}{8}$

左図の斜線部をx軸のまわりに回転させた立体の体積は

$$\int_{\frac{7}{8}}^{1}\pi(1-x^2)dx+\int_{\frac{1}{8}}^{\frac{1}{2}}\pi\left(\frac{1}{4}-x^2\right)dx$$

$$=\pi\left[-\frac{x^3}{3}+x\right]_{\frac{7}{8}}^{1}+\pi\left[-\frac{x^3}{3}+\frac{1}{4}x\right]_{\frac{1}{8}}^{\frac{1}{2}}$$

$$=\pi\left(-\frac{1}{3}+1+\frac{7\cdot7\cdot7}{3\cdot8\cdot8\cdot8}-\frac{7}{8}-\frac{1}{24}+\frac{1}{8}+\frac{1}{3\cdot8\cdot8\cdot8}-\frac{1}{32}\right)$$

$$=\pi\left(\frac{-32+96-84-4+12-3}{96}+\frac{344}{1536}\right)$$

$$=\pi\left(\frac{-15}{96}+\frac{43}{192}\right)=\frac{13}{192}\pi$$

よって $V_n=\frac{a^3}{8^n}\frac{13\pi}{192}=\frac{13a^3\pi}{3\cdot8^{n+2}}$

(ii) $V_m=\frac{13a^3\pi}{192}\frac{\frac{1}{8}\{1-(\frac{1}{8})^m\}}{1-\frac{1}{8}}$

$$V=\lim_{m\to\infty}V_m=\frac{13a^3\pi}{192}\frac{1}{7}=\frac{13a^3\pi}{1344}$$

$192 \times 7 = 1344$

24×64: 96, 144, 1536

$49 \times 7 = 343$

$123 - 108 = 15$

$344 \div 2 = 172$, $172 \div 2 = 86$, $86 \div 2 = 43$

$1536 \div 8 = 192$