



$$V = 2 \int_0^r \pi x^2 dx + \pi r^2 l = 2\pi \left[\frac{x^3}{3} \right]_0^r + \pi r^2 l = \frac{2}{3} \pi r^3 + \pi r^2 l$$

$$l = -r^2 + C, \quad 0 \leq r^2 \leq C \text{ かつ } 0 \leq r \leq \sqrt{C} \quad \text{--- ①}$$

$$V = \frac{2}{3} \pi r^3 + \pi r^2 (-r^2 + C) = -\pi r^4 + \frac{2}{3} \pi r^3 + \pi C r^2$$

$$\frac{dV}{dr} = -4\pi r^3 + 2\pi r^2 + 2\pi C r = 2\pi r^2 (-2r + r + C) = 2\pi r^2 (-r + C)$$

$$\frac{dV}{dr} = 0 \text{ のとき } r = 0, \frac{1 \pm \sqrt{1+24C}}{6}$$

Vの増減表は左表のようになります

r	0	...	$\frac{1+\sqrt{1+24C}}{6}$...
$\frac{dV}{dr}$	0	+	0	-
V		↗	最大	↘

$$\frac{1+\sqrt{1+24C}}{6} \leq \sqrt{C} \text{ のとき } 1+\sqrt{1+24C} \leq 6\sqrt{C}, \sqrt{1+24C} \leq 6\sqrt{C}-1$$

$$1+24C \leq 36C - 12\sqrt{C} + 1, \sqrt{C} \leq C, 1 \leq \sqrt{C}, C \geq 1 \quad \text{--- ②}$$

①, ② かつ. $C \leq 1$ のとき, $r = \sqrt{C}, l = 0.$

$$C \geq 1 \text{ のとき, } r = \frac{1+\sqrt{1+24C}}{6}, l = -\frac{1+2\sqrt{1+24C}+1+24C}{36} + C = \frac{12C - 2\sqrt{1+24C} - 2}{36} = \frac{6C - \sqrt{1+24C} - 1}{18}$$