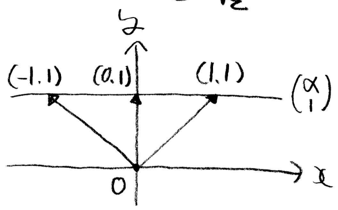
東大理科　1980前期　③

(1) $A=\sqrt{2}\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}=\sqrt{2}\begin{pmatrix}\cos\frac{\pi}{4} & -\sin\frac{\pi}{4}\\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4}\end{pmatrix}$　Aは原点を中心に$\frac{\pi}{4}$回転させる行列



$\alpha=-1,\ 0,\ 1$

(2) $A^n=\sqrt{2}^n\begin{pmatrix}\cos\frac{n\pi}{4} & -\sin\frac{n\pi}{4}\\ \sin\frac{n\pi}{4} & \cos\frac{n\pi}{4}\end{pmatrix}$　$\begin{pmatrix}p_n\\q_n\end{pmatrix}=\sqrt{2}^n\begin{pmatrix}\cos\frac{n\pi}{4} & -\sin\frac{n\pi}{4}\\ \sin\frac{n\pi}{4} & \cos\frac{n\pi}{4}\end{pmatrix}\begin{pmatrix}\alpha\\1\end{pmatrix}=\sqrt{2}^n\begin{pmatrix}\cos\frac{n\pi}{4}\cdot\alpha-\sin\frac{n\pi}{4}\\ \sin\frac{n\pi}{4}\cdot\alpha+\cos\frac{n\pi}{4}\end{pmatrix}$

$$a_n=\frac{\cos\frac{n\pi}{4}\cdot\alpha-\sin\frac{n\pi}{4}}{\sin\frac{n\pi}{4}\cdot\alpha+\cos\frac{n\pi}{4}}=\frac{\cos\frac{n\pi}{4}\frac{\alpha}{\sqrt{\alpha^2+1}}-\sin\frac{n\pi}{4}\frac{1}{\sqrt{\alpha^2+1}}}{\sin\frac{n\pi}{4}\frac{\alpha}{\sqrt{\alpha^2+1}}+\cos\frac{n\pi}{4}\frac{1}{\sqrt{\alpha^2+1}}}=\frac{\cos\frac{n\pi}{4}\cos\varphi-\sin\frac{n\pi}{4}\sin\varphi}{\sin\frac{n\pi}{4}\cos\varphi+\cos\frac{n\pi}{4}\sin\varphi}=\frac{\cos\left(\frac{n\pi}{4}+\alpha\right)}{\sin\left(\frac{n\pi}{4}+\alpha\right)}=\frac{1}{\tan\left(\frac{n\pi}{4}+\alpha\right)}$$

※ φは $\cos\varphi=\frac{\alpha}{\sqrt{\alpha^2+1}}$, $\sin\varphi=\frac{1}{\sqrt{\alpha^2+1}}$ を満たす値　　よって、4個