

(1) 1の目のあつ面を黒い面を表す

1の目が 上面に あつとき								$\frac{2}{6}$ の上面, $\frac{4}{6}$ の側面
1の目が 側面に あつとき								$\frac{1}{6}$ の上面, $\frac{4}{6}$ の側面, $\frac{1}{6}$ の底面
1の目が 底面に あつとき								$\frac{4}{6}$ の側面, $\frac{2}{6}$ の底面

上記より. $p_1 = \frac{2}{6} = \frac{1}{3}$, $q_1 = \frac{4}{6} = \frac{2}{3}$, $r_1 = 0$

(2) 上記より.

$$p_n = \frac{2}{6}p_{n-1} + \frac{1}{6}q_{n-1} = \frac{1}{3}p_{n-1} + \frac{1}{6}q_{n-1} \quad \text{--- (1)}$$

$$q_n = \frac{4}{6}p_{n-1} + \frac{4}{6}q_{n-1} + \frac{4}{6}r_{n-1} = \frac{2}{3}p_{n-1} + \frac{2}{3}q_{n-1} + \frac{2}{3}r_{n-1} \quad \text{--- (2)}$$

$$r_n = \frac{1}{6}q_{n-1} + \frac{2}{6}r_{n-1} = \frac{1}{6}q_{n-1} + \frac{1}{3}r_{n-1} \quad \text{--- (3)}$$

(3) ②より. $p_{n-1} + q_{n-1} + r_{n-1} = 1$ より. $q_n = \frac{2}{3}$

①より. $p_n = \frac{1}{3}p_{n-1} + \frac{1}{6}$ $x = \frac{1}{3}x + \frac{1}{6}$ $\frac{2}{3}x = \frac{1}{6}$ $x = \frac{1}{6}$

$$p_n - \frac{1}{6} = \frac{1}{3}(p_{n-1} - \frac{1}{6}) = (\frac{1}{3})^2(p_{n-2} - \frac{1}{6}) = \dots = (\frac{1}{3})^{n-1}(p_1 - \frac{1}{6}) = (\frac{1}{3})^{n-1} \frac{1}{6}$$

$$p_n = \frac{1}{6} + \frac{1}{2}(\frac{1}{3})^n$$

③より. $r_n = \frac{1}{3}r_{n-1} + \frac{1}{6}$

$$r_n - \frac{1}{6} = \frac{1}{3}(r_{n-1} - \frac{1}{6}) = (\frac{1}{3})^2(r_{n-2} - \frac{1}{6}) = \dots = (\frac{1}{3})^{n-1}(r_1 - \frac{1}{6}) = -\frac{1}{6}(\frac{1}{3})^{n-1}$$

$$r_n = \frac{1}{6} - \frac{1}{2}(\frac{1}{3})^n$$

よって. $p = \frac{1}{6}$, $q = \frac{2}{3}$, $r = \frac{1}{6}$