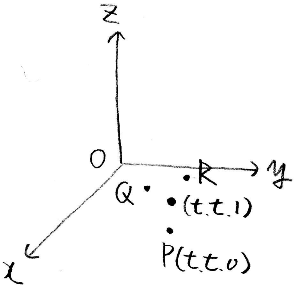
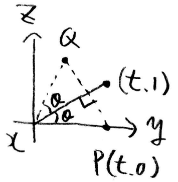
東大理科　1985前期　③



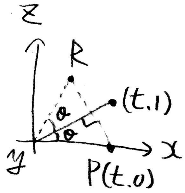


左図より $\begin{pmatrix}\cos2\theta & -\sin2\theta\\ \sin2\theta & \cos2\theta\end{pmatrix}\begin{pmatrix}t\\0\end{pmatrix} = \begin{pmatrix}\frac{t^2-1}{t^2+1} & -\frac{2t}{t^2+1}\\ \frac{2t}{t^2+1} & \frac{t^2-1}{t^2+1}\end{pmatrix}\begin{pmatrix}t\\0\end{pmatrix} = \begin{pmatrix}\frac{t^3-t}{t^2+1}\\ \frac{2t^2}{t^2+1}\end{pmatrix}$ より

※ $\cos\theta = \frac{t}{\sqrt{t^2+1}}$, $\sin\theta = \frac{1}{\sqrt{t^2+1}}$ より

$\cos2\theta = \cos^2\theta - \sin^2\theta = \frac{t^2}{t^2+1} - \frac{1}{t^2+1} = \frac{t^2-1}{t^2+1}$, $\sin2\theta = 2\sin\theta\cos\theta = \frac{2t}{t^2+1}$

Qの座標は $\left(t, \frac{t^3-t}{t^2+1}, \frac{2t^2}{t^2+1}\right)$



左図より 上記と同様にして、

Rの座標は $\left(\frac{t^3-t}{t^2+1}, t, \frac{2t^2}{t^2+1}\right)$

Q、Rの中点Sの座標は

$\frac{1}{2}\left(t+\frac{t^3-t}{t^2+1}, \frac{t^3-t}{t^2+1}+t, \frac{4t^2}{t^2+1}\right) = \frac{1}{2}\left(\frac{t^3\cancel{+t}+t^3\cancel{-t}}{t^2+1}, \frac{t^3\cancel{-t}+t^3\cancel{+t}}{t^2+1}, \frac{4t^2}{t^2+1}\right) = \left(\frac{t^3}{t^2+1}, \frac{t^3}{t^2+1}, \frac{2t^2}{t^2+1}\right)$

O、P、Sを含む平面と直線QRは直交する。

△OPSの面積は $\sqrt{2}t\,\frac{\cancel{2}t^2}{t^2+1}\,\frac{1}{\cancel{2}} = \frac{\sqrt{2}t^3}{t^2+1}$ —①

QRの長さは $\sqrt{2\left(\frac{t^3-t}{t^2+1}-t\right)^2} = \sqrt{2\left(\frac{\cancel{t^3}-t-\cancel{t^3}-t}{t^2+1}\right)^2} = \frac{2\sqrt{2}t}{t^2+1}$ —②

①②より求める体積は $\frac{\sqrt{2}t^3}{t^2+1}\,\frac{2\sqrt{2}t}{t^2+1}\,\frac{1}{3} = \frac{4t^4}{3(t^2+1)^2}$