

$$\frac{-2+3}{72}$$

$$(1) f(x) = x^4 + x^3 + \frac{1}{4}x^2 + \frac{1}{4}(x^2 + \frac{2}{3}x + \frac{1}{9}) - \frac{1}{36} + \frac{1}{24}$$

$$= (x^2 + \frac{1}{2}x)^2 + \frac{1}{4}(x + \frac{1}{3})^2 + \frac{1}{72} > 0$$

$$(2) g(-1) = -1 + 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{24} + \frac{1}{120} = \frac{-60 + 20 - 5 + 1}{120} = -\frac{44}{120} < 0 \quad \text{--- (1)}$$

$$g(0) = \frac{1}{120} > 0 \quad \text{--- (2)}$$

$$g'(x) = 5x^4 + 4x^3 + \frac{3}{2}x^2 + \frac{1}{3}x + \frac{1}{24}$$

$$= 5(x^4 + \frac{4}{5}x^3 + \frac{4}{25}x^2) - \frac{4}{5}x^2 + \frac{3}{2}x^2 + \frac{1}{3}x + \frac{1}{24}$$

$$= 5(x + \frac{2}{5}x)^2 + \frac{7}{10}(x^2 + \frac{10}{21}x + \frac{25}{441}) - \frac{5}{2} \cdot \frac{1}{63} + \frac{1}{24}$$

$$= 5(x + \frac{2}{5}x)^2 + \frac{7}{10}(x + \frac{5}{21})^2 + \frac{-5 \cdot 12 + 63}{2 \cdot 63 \cdot 12}$$

$$= 5(x + \frac{2}{5}x)^2 + \frac{7}{10}(x + \frac{5}{21})^2 + \frac{3}{1512} > 0$$

$$\begin{array}{r} 21 \\ 21 \\ 42 \\ 99 \end{array} \quad \begin{array}{r} 63 \\ 29 \\ 25 \\ 126 \\ 1512 \end{array}$$

∴ g'(x) は単調増加 --- (3)

①②③より題意は言証明された。