東大理科　1997前期　②

$m^2-(a-1)m+\frac{n^2}{2n+1}a>0$ が成り立つとき．$m^2+m>\left(m-\frac{n^2}{2n+1}\right)a$

$m>\frac{n^2}{2n+1}$ のとき $\frac{m^2+m}{m-\frac{n^2}{2n+1}}>a$，$m<\frac{n^2}{2n+1}$ のとき $\frac{m^2+m}{m-\frac{n^2}{2n+1}}<a$ が成り立つ ―①

$f(m)=\frac{m^2+m}{m-\frac{n^2}{2n+1}}$ $\left(m\neq\frac{n^2}{2n+1}\right)$ とする．

$$f'(m)=\frac{(2m+1)\left(m-\frac{n^2}{2n+1}\right)-m^2-m}{\left(m-\frac{n^2}{2n+1}\right)^2}=\frac{2m^2-\frac{2n^2}{2n+1}m+\cancel{m}-\frac{n^2}{2n+1}-m^2-\cancel{m}}{\left(m-\frac{n^2}{2n+1}\right)^2}=\frac{m^2-\frac{2n^2}{2n+1}m-\frac{n^2}{2n+1}}{\left(m-\frac{n^2}{2n+1}\right)^2}$$

$f'(m)=0$ のとき $m=\frac{n^2}{2n+1}\pm\sqrt{\frac{n^4}{(2n+1)^2}+\frac{n^2}{2n+1}}=\frac{n^2}{2n+1}\pm n\sqrt{\frac{n^2+2n+1}{(2n+1)^2}}=\frac{n^2}{2n+1}\pm\frac{n(n+1)}{2n+1}$

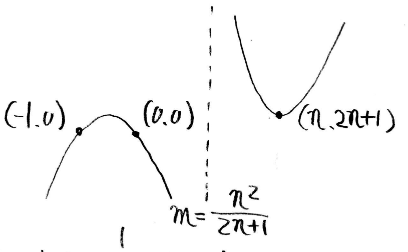
$=\frac{n^2\pm(n^2+n)}{2n+1}=n,\ -\frac{1}{2+\frac{1}{n}}$

m	$-\infty$	$\cdots$	$-\frac{1}{2+\frac{1}{n}}$	$\cdots$	$\frac{n^2}{2n+1}$		$\cdots$	n	$\cdots$	∞
$f'(m)$		$+$	0	$-$			$-$	0	$+$	
$f(m)$	$-\infty$	$\nearrow$	極大	$\searrow$	$-\infty$	∞	$\searrow$	$2n+1$	$\nearrow$	∞

$f(m)$ の増減表は左表のようになる．

※ $f(n)=\frac{n+1}{1-\frac{n}{2n+1}}=\frac{(2n+1)\cancel{(n+1)}}{\cancel{2n+1-n}}=2n+1$

$\lim_{m\to-\infty}\frac{m+1}{1-\frac{\frac{n^2}{2n+1}}{m}}=-\infty$，$\lim_{m\to\infty}\frac{m+1}{1-\frac{\frac{n^2}{2n+1}}{m}}=\infty$，$\lim_{m\to\frac{n^2}{2n+1}-0}\frac{m^2+m}{m-\frac{n^2}{2n+1}}=-\infty$，$\lim_{m\to\frac{n^2}{2n+1}+0}\frac{m^2+m}{m-\frac{n^2}{2n+1}}=\infty$



$f(m)$ のグラフは左図のようになる ―②

※ $-1<-\frac{1}{2+\frac{1}{n}}<0$．$f(-1)=0$、$f(0)=0$

$m=\frac{n^2}{2n+1}$ のとき．$m^2-(a-1)m+\frac{n^2}{2n+1}a=\frac{n^4}{(2n+1)^2}-(a-1)\frac{n^2}{2n+1}+\frac{n^2}{2n+1}a=\frac{n^2}{2n+1}\left(\frac{n^2}{2n+1}-\cancel{a}+1+\cancel{a}\right)>0$ ―③

①②③より　$0<a<2n+1$