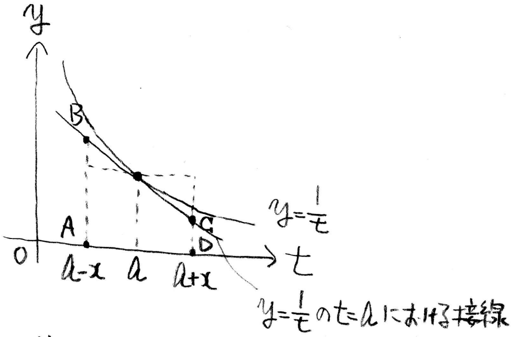
東大理科　2007前期　⑥

(1)

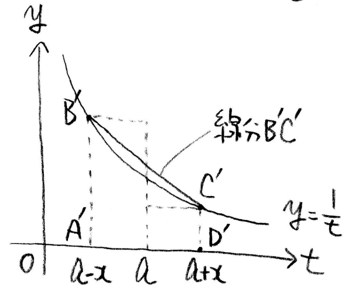


$y=\frac{1}{t}$, $y'=-\frac{1}{t^2}$, $y''=\frac{2}{t^3}$ より, $y=\frac{1}{t}$ は下に凸であるから,

$y=\frac{1}{t}$ の $t=a$ における接線は, $y=\frac{1}{t}$ の下側にある.

左図のように A, B, C, D をとると,

$\frac{2x}{a}$ = 台形ABCDの面積 $< \int_{a-x}^{a+x} \frac{1}{t}dt$ —①



左図のように A′, B′, C′, D′ をとると,

$x\left(\frac{1}{a+x}+\frac{1}{a-x}\right)$ = 台形A′B′C′D′の面積 $> \int_{a-x}^{a+x} \frac{1}{t}dt$ —②

①②より, 題意は示された

(2) $\int_{a-x}^{a+x} \frac{1}{t}dt = \left[\log t\right]_{a-x}^{a+x} = \log(a+x) - \log(a-x) = \log\frac{a+x}{a-x}$

$\frac{a+x}{a-x} = \sqrt{2}$ とすると, $a+x = \sqrt{2}a - \sqrt{2}x$. $x = \frac{\sqrt{2}-1}{\sqrt{2}+1}a = \frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}a = (3-2\sqrt{2})a$

$\frac{2x}{a} = 6-4\sqrt{2}$

$\int_{a-x}^{a+x} \frac{1}{t}dt = \frac{1}{2}\log 2$

$x\left(\frac{1}{a+x}+\frac{1}{a-x}\right) = (3-2\sqrt{2})\left(\frac{1}{4-2\sqrt{2}}+\frac{1}{2\sqrt{2}-2}\right) = \frac{3-2\sqrt{2}}{2}\,\frac{\cancel{\sqrt{2}}-1+2-\cancel{\sqrt{2}}}{(2-\sqrt{2})(\sqrt{2}-1)} = \frac{3-2\sqrt{2}}{2}\,\frac{1}{2\sqrt{2}-2-2+\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{2}\,\frac{3\sqrt{2}+4}{(3\sqrt{2}-4)(3\sqrt{2}+4)} = \frac{9\sqrt{2}+\cancel{12}-\cancel{12}-8\sqrt{2}}{2(18-16)} = \frac{\sqrt{2}}{4}$

(1)より, $6-4\sqrt{2} < \frac{1}{2}\log 2 < \frac{\sqrt{2}}{4}$, $12-8\sqrt{2} < \log 2 < \frac{\sqrt{2}}{2}$

ここで $12-8\sqrt{2} > 12-8\times 1.415 = 0.68$, $\frac{\sqrt{2}}{2} < \frac{1.42}{2} = 0.71$ より,

$$\begin{array}{r} 1.415 \\ \times\quad 8 \\ \hline 11.320 \end{array}$$

$0.68 < \log 2 < 0.71$