東大文科　2010前期　1

(1)

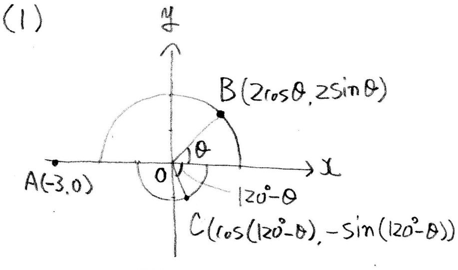
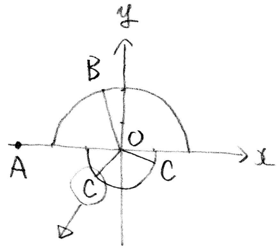


図 1



△ABCはOを含むから
このCは不適

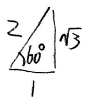


図1より Bの座標は $(2\cos\theta,\ 2\sin\theta)$

Cの座標は $(\cos(120^\circ-\theta),\ -\sin(120^\circ-\theta)) = (\cos120^\circ\cos\theta+\sin120^\circ\sin\theta,\ -\sin120^\circ\cos\theta+\cos120^\circ\sin\theta)$

$= \left(-\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta,\ -\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)$

△OABの面積は $3\cdot\cancel{2}\sin\theta\cdot\frac{1}{\cancel{2}} = 3\sin\theta$

△OACの面積は $3\left(\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta\right)\frac{1}{2} = \frac{3}{4}\sin\theta+\frac{3\sqrt{3}}{4}\cos\theta$

よって $3\sin\theta = \frac{3}{4}\sin\theta+\frac{3\sqrt{3}}{4}\cos\theta$, $\frac{9}{\cancel{4}}\sin\theta = \frac{3\sqrt{3}}{\cancel{4}}\cos\theta$ $\tan\theta = \frac{1}{\sqrt{3}}$. $0^\circ<\theta<120^\circ$ より $\theta = 30^\circ$

(2) △OABと△OACの面積の和を $f(\theta)$ とすると

$f(\theta) = 3\sin\theta+\frac{3}{4}\sin\theta+\frac{3\sqrt{3}}{4}\cos\theta = \frac{15}{4}\sin\theta+\frac{3\sqrt{3}}{4}\cos\theta = \frac{3\sqrt{7}}{2}\ \dfrac{\frac{15}{4}\sin\theta+\frac{3\sqrt{3}}{4}\cos\theta}{\frac{3\sqrt{7}}{2}}$

$$\begin{array}{r|l} 2 & 252 \\ 2 & 126 \\ 3 & 63 \\ 3 & 21 \\ & 7 \end{array}$$

※ $\sqrt{\frac{225}{16}+\frac{27}{16}} = \sqrt{\frac{\overset{63}{\cancel{252}}}{\underset{4}{\cancel{16}}}} = \frac{3\sqrt{7}}{2}$

$= \frac{3\sqrt{7}}{2}\left(\frac{\cancel{2}\sqrt{7}}{\underset{7}{\cancel{21}}}\ \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{4}}}\sin\theta+\frac{\cancel{2}\sqrt{7}}{\underset{7}{\cancel{21}}}\ \frac{3\sqrt{3}}{\underset{2}{\cancel{4}}}\cos\theta\right) = \frac{3\sqrt{7}}{2}\left(\frac{5\sqrt{7}}{14}\sin\theta+\frac{\sqrt{21}}{14}\cos\theta\right)$

ここで α を $0^\circ<\alpha<90^\circ$, $\sin\alpha = \frac{\sqrt{21}}{14}$, $\cos\alpha = \frac{5\sqrt{7}}{14}$ を満たす値とすると

$f(\theta) = \frac{3\sqrt{7}}{2}\sin(\theta+\alpha)$

よって $f(\theta)$ の最大値は $\theta+\alpha = 90^\circ$ のとき $\frac{3\sqrt{7}}{2}$

このとき $\sin\theta = \sin(90^\circ-\alpha) = \cos\alpha = \frac{5\sqrt{7}}{14}$