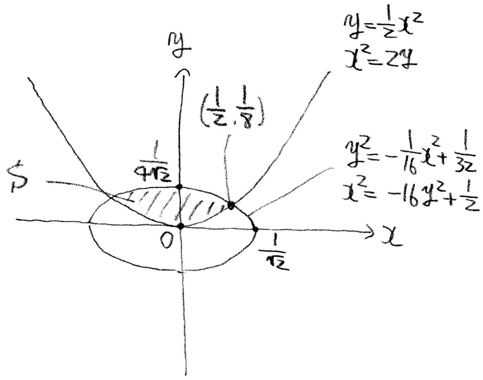
東大理科　2012前期 ③

(1)



Sは左図の斜線部，境界線上の点含む

よって

$$V_1=2\int_0^{\frac{1}{2}}\left\{\pi\left(-\frac{1}{16}x^2+\frac{1}{32}\right)-\pi\frac{1}{4}x^4\right\}dx$$

$$=\frac{\pi}{16}\int_0^{\frac{1}{2}}(-8x^4-2x^2+1)dx$$

$$=\frac{\pi}{16}\left[-8\frac{x^5}{5}-2\frac{x^3}{3}+x\right]_0^{\frac{1}{2}}$$

$$=\frac{\pi}{16}\left(-\frac{8}{5}\frac{1}{32}-\frac{2}{3}\frac{1}{8}+\frac{1}{2}\right)$$

$$=\frac{\pi}{16}\left(-\frac{1}{20}-\frac{1}{12}+\frac{1}{2}\right)=\frac{\pi}{32}\left(-\frac{1}{10}-\frac{1}{6}+1\right)$$

$$=\frac{\pi}{32}\frac{-3-5+30}{30}=\frac{\pi}{32}\frac{\cancel{22}^{11}}{\cancel{30}_{15}}=\frac{11}{480}\pi$$

$$\begin{array}{r} 32 \\ \times 15 \\ \hline 160 \\ 32 \\ \hline 480 \end{array}$$

※ $y=\frac{1}{2}x^2$ と $\frac{x^2}{4}+4y^2=\frac{1}{8}$ の交点は

$\frac{x^2}{4}+4\frac{1}{4}x^4=\frac{1}{8}$　$8x^4+2x^2-1=0$．$x^2=\frac{-1\pm\sqrt{1+8}}{8}=\frac{-1\pm3}{8}=-\frac{1}{2},\frac{1}{4}$

$x^2\geqq0$ より $x^2=\frac{1}{4}$．$x=\pm\frac{1}{2}$．$y=\frac{1}{8}$ より $\left(\pm\frac{1}{2},\frac{1}{8}\right)$

$$V_2=\int_0^{\frac{1}{8}}\pi\cdot2y\,dy+\int_{\frac{1}{8}}^{\frac{1}{4\sqrt{2}}}\pi\left(-16y^2+\frac{1}{2}\right)dy=\cancel{2}\pi\left[\frac{y^2}{\cancel{2}}\right]_0^{\frac{1}{8}}+\pi\left[-16\frac{y^3}{3}+\frac{1}{2}y\right]_{\frac{1}{8}}^{\frac{1}{4\sqrt{2}}}$$

$$=\frac{1}{64}\pi+\pi\left(-\frac{16}{3}\frac{1}{\underset{4}{\cancel{64}}\cdot2\sqrt{2}}+\frac{1}{8\sqrt{2}}+\frac{16}{3}\frac{1}{\underset{4}{\cancel{64}}\cdot8}-\frac{1}{2}\frac{1}{8}\right)=\frac{1}{64}\pi+\pi\left(-\frac{1}{24\sqrt{2}}+\frac{3}{24\sqrt{2}}+\frac{1}{96}-\frac{6}{96}\right)$$

$$\begin{array}{r} 8 \\ 24\overline{)192} \\ 192 \\ \hline 0 \end{array}$$

$$=\frac{1}{64}\pi+\pi\left(\frac{1}{12\sqrt{2}}-\frac{5}{96}\right)=\frac{3-10}{192}\pi+\frac{\sqrt{2}}{24}\pi=\frac{8\sqrt{2}-7}{192}\pi$$

(2)
$$\frac{V_2}{V_1}=\frac{\frac{8\sqrt{2}-7}{192}\cancel{\pi}}{\frac{11}{480}\cancel{\pi}}=\frac{8\sqrt{2}-7}{\underset{2}{\cancel{192}}}\frac{\overset{5}{\cancel{480}}}{11}=\frac{40\sqrt{2}-35}{22}<\frac{40\times1.42-35}{22}=\frac{21.8}{22}<1$$

$$\begin{array}{r} 1.42 \\ \times\quad 40 \\ \hline 56.8\cancel{0} \end{array}$$